MATH 206, Section A

Test # 1

1. (20 points) Given a rectangular point, change it to cylindrical **and** spherical coordinates.

$$\left(-\sqrt{3}, -3, -2\right)$$

2. (20 points) Let

$$\begin{aligned} \alpha_1 &: \quad x - 2y + z = 1, \\ \alpha_2 &: \quad 2x + y + z = 7, \text{ and} \\ \alpha_3 &: \quad x - 5y + 3z = 4. \end{aligned}$$

Let ℓ be the line of intersection of α_1 and α_2. Find parametric equations of ℓ **and** the angle between ℓ and α_3.

3. (20 points) Find a point on the curve

$$\mathbf{r}(t) = \left\langle 2\ln t, 4\arctan t, t^2 \right\rangle$$

such that the tangent line at the point is perpendicular to the plane

$$\langle x, y, z\rangle = \langle 1, 0, 3\rangle + t\langle 1, 0, -1\rangle + s\langle 0, 1, -1\rangle .$$

Find the distance between the point and the plane.

4. (20 points) Find the domain of the given function. Sketch the domain. Is the domain open, closed, or neither? Explain your answer.

$$f(x, y) = \sqrt{y - \ln(-x)} \cdot \ln(-y + \arccos x)$$

5. (20 points) For the given vector-function of three variables

$$\mathbf{F}(x, y, z) = \left\langle \ln\left(1 + (xy)^2\right), \arctan\left(xz^2\right) \right\rangle$$

find its (total) derivative, the Jacobian matrix at $(-1, 2, 1)$, and the image of the vector $\langle 10, 5, 7\rangle$ under the corresponding linear transformation.

6. (20 points) Let $F = F(u, v)$ be a function of two variables. Suppose that $u = x + y$ and $v = xy$. Find $\dfrac{\partial^2 F}{\partial x \partial y}$.