

1. Suppose $T : \mathbf{R}^n \rightarrow \mathbf{R}^m$ is a transformation. What are the two defining properties that T must satisfy in order to be a *linear* transformation?

2. Suppose that $T \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} -4 \\ -3 \\ -1 \end{bmatrix}$ and $T \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$.

2A. Find the (standard) matrix for T .

2B. Find $T \left(\begin{bmatrix} 2 \\ 5 \end{bmatrix} \right)$.

3. Suppose $T : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ has (standard) matrix $A = \begin{bmatrix} 2 & 10 & 7 \\ 1 & 5 & 6 \\ -2 & -10 & -15 \end{bmatrix}$

3A. What, if any, conditions does a vector $\mathbf{b} \in \mathbf{R}^3$ need to satisfy in order to be in the image of T ? Show any RREF matrices you use in finding your answer.

3B. Show that $\mathbf{b} = \begin{bmatrix} 15 \\ -5 \\ 25 \end{bmatrix}$ does indeed satisfy the answer to 3A.

3C. Find just one $\mathbf{x} \in \mathbf{R}^3$ such that $T(\mathbf{x}) = \begin{bmatrix} 15 \\ -5 \\ 25 \end{bmatrix}$.