

Name _____

Quiz #7
Mathematics 309a Abstract Algebra
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Let G be a group and let $a \in G$ be a fixed element. Define $\rho_a : G \rightarrow G$ by the rule $\rho_a(g) = ga \ \forall g \in G$, i.e. ρ_a multiplies what goes into it on the right by a . Construct one of these ρ 's for each element of G and define $K = \{ \rho_a : a \in G \}$ to be the set of all these ρ 's.

A. Let $x \in G$ be arbitrary. Write an element of G that ρ_a sends to x : _____.

B. What does part A prove about ρ_a ? ρ_a is _____.

C. Let $x, y \in G$ and $\rho_a(x) = \rho_a(y)$. Prove that $x = y$.

D. What does part C prove about ρ_a ? ρ_a is _____.

E. What do we call a function from G to G that has the properties from parts B and D?

A _____ of G .

F. If ρ_a is composed with ρ_b then the function $\rho_a \circ \rho_b$ is an element of K , i.e. equals ρ_c for some $c \in G$. The c that gives $\rho_a \circ \rho_b$ is _____.

G. If e is the identity element of G , then ρ_e is the identity permutation of G . Prove this fact:

_____.

H. The answer to part E assures us that ρ_a has an inverse for every $a \in G$, i.e. is of the form ρ_c . The c that gives the inverse for ρ_a is _____.

I. We call a set with an operation that satisfies F, G, and H a _____.