1. Let $C = \begin{bmatrix} 5 & -10 & 7 & 7 \\ -3 & 6 & -8 & 11 \\ 7 & -14 & 9 & 13 \end{bmatrix}$ and label the column vectors as $\mathbf{c}_1$, $\mathbf{c}_2$, $\mathbf{c}_3$ and $\mathbf{c}_4$, respectively.

1A) Let $\mathbf{b}$ be this linear combination of the column vectors:

$$\mathbf{b} = 4\mathbf{c}_1 + 3\mathbf{c}_2 + 2\mathbf{c}_3 + \mathbf{c}_4.$$

Explicitly, what is $\mathbf{b}$? (Reminder: you know how to find this LC using two matrices on your calculator).

your answer: $\mathbf{b} = ???$

1B) Find all solutions of $C\mathbf{x} = \mathbf{b}$; write your answer in the form $\mathbf{p}+\mathbf{v}_h$ where $\mathbf{p}$ is a particular solution of $C\mathbf{x} = \mathbf{b}$ obtained from the RREF of the appropriate augmented matrix, and $\mathbf{v}_h$ is the set of all solutions to the homogeneous equation $C\mathbf{x} = \mathbf{0}$. Give the RREF'd matrix you used to find the solutions.

1C) The statement "Let $\mathbf{b}$ be this linear combination..." at the start of this problem says that $\mathbf{x} = \mathbf{q}$ is a solution of the matrix equation $C\mathbf{x} = \mathbf{b}$ for what particular solution $\mathbf{q}$?

your answer: $\mathbf{q} = ???$

1D) In the solution found in (1B), what values do the free variables need to be assigned to satisfy $\mathbf{q} = \mathbf{p} + \mathbf{v}_h$?

1E) Set all the free variables in $\mathbf{p} + \mathbf{v}_h$ to 10. What (new) particular solution do you get (call it $\mathbf{w}$).

1F) Find $C\mathbf{w}$ (hopefully the result isn't a surprise).

1G) The solution set of $C\mathbf{x} = \mathbf{b}$ can also be written as $\mathbf{q} + \mathbf{v}_h$. What values must the free variables have to have in order to satisfy $\mathbf{w} = \mathbf{q} + \mathbf{v}_h$?

2. Let $A = \begin{bmatrix} 5 & 1 & 7 & 13 & 13 \\ 3 & 1 & 5 & 9 & 5 \\ 4 & 1 & 6 & 11 & 9 \\ 2 & 1 & 4 & 7 & 1 \end{bmatrix}$. Let $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$, let $\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ 4 \\ 10 \end{bmatrix}$ and let $\mathbf{v} = \begin{bmatrix} 21 \\ 3 \\ v_3 \\ v_4 \end{bmatrix}$.

Label the column vectors of A as $\mathbf{a}_1, \ldots, \mathbf{a}_5$.

It's a fact that $\text{RREF}\left(\left[\begin{array}{c|cccc} A & 1 & 0 & 0 & 0 \\ & 0 & 1 & 0 & 0 \\ & 0 & 0 & 1 & 0 \\ & 0 & 0 & 0 & 1 \end{array}\right]\right)$ is $\left[\begin{array}{ccccc|cccc} 1 & 0 & 1 & 2 & 4 & 0 & 0 & 1/2 & -1/2 \\ 0 & 1 & 2 & 3 & -7 & 0 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -3/2 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1/2 & -1/2 \end{array}\right]$.

2A) Find all conditions on $b_1, \ldots, b_4$ that make the equation $A\mathbf{x} = \mathbf{b}$ consistent.

2B) Use the answer to 2A to find m_1 and m_2 for which $A\mathbf{x} = \mathbf{m}$ has a solution. Show the work and label the results.

2C) Now find all solutions of $A\mathbf{x} = \mathbf{m}$ in parametric vector form.

2D) Use the answer in 2A to find all values of v_3 and v_4 for which $\mathbf{v}$ is in Span $\{\mathbf{a}_1, \ldots, \mathbf{a}_5\}$.

2E) Does Span $\{\mathbf{a}_1, \ldots, \mathbf{a}_5\} = \mathbb{R}^4$? Explain! If the answer is "no", give a specific vector supporting this answer.

3A. Suppose $\mathbf{v}_1, \ldots, \mathbf{v}_p$ are p vectors in $\mathbb{R}^m$ for some m. Give the in-class definition of what it means for the set $\{\mathbf{v}_1, \ldots, \mathbf{v}_p\}$ to be *linearly independent.*

3B. Let $\mathbf{a}_1, \ldots, \mathbf{a}_5$ be as in question (**2**) on the previous page. Is $\{\mathbf{a}_1, \ldots, \mathbf{a}_5\}$ a linearly independent set? Answer in terms of the definition in (3A).

4. Suppose $\mathbf{u}_1, \ldots, \mathbf{u}_6$ are six different vectors in $\mathbb{R}^m$ for some m, and that $4\mathbf{u}_1 - 8\mathbf{u}_2 + 0\mathbf{u}_3 + 2\mathbf{u}_4 - 4\mathbf{u}_5 + 0\mathbf{u}_6 = \mathbf{0}$.

4a) Use the equation in (**4**) to write $\mathbf{u}_5$ as a linear combination of the other five vectors in $\{\mathbf{u}_1, \ldots, \mathbf{u}_6\}$ or explain why this can't be done knowing only this equation.

4b) Does the equation in (**4**) necessarily mean that $\mathbf{u}_3$ cannot be written as a linear combination of the other five vectors in $\{\mathbf{u}_1, \ldots, \mathbf{u}_6\}$? Explain.

5.) Suppose that $\{\mathbf{v}_1, \ldots, \mathbf{v}_7\}$ is linearly independent set. Explain why you cannot write $\mathbf{v}_7$ as a linear combination

$$\mathbf{v}_7 = s_1\mathbf{v}_1 + \cdots + s_6\mathbf{v}_6$$

of the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_6$.

6. In a certain Exchange Model economy, there are four sectors labeled A, B, C and D.
Sector B consumes 10% of every sector's output, including its own.
Sector B is the only sector that consumes any of its own output.
Sectors C and A each consume 80% of each other's output.
Sectors C and D consume 3/4 and 1/20 of B's output, respectively.
Sector C consumes half of D's output.
All output of every sector is consumed by some sector.

6A) Fill out the exchange table for this economy.

6B) What is the *third* equation in the system of equations you need to set up to find the equilibrium prices P_A, P_B, P_C and P_D.

6C) Solve the system discussed in 6B to find the equilibrium prices. Show any RREF'd matrices you use.

6D) Rank the sectors in order of decreasing equilibrium prices.