

MATH 205A,B LINEAR ALGEBRA - PROF. P. WONG

EXAM I - FEBRUARY 6, 2013

NAME: _____ Section:(Circle one) A(1 : 10) B(2 : 40)

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	Maximum Score	Your Score
1.	20	
2.	20	
3.	20	
4.	20	
5.	20	
Total	100	

1. (a) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a transformation given by

$$T(x, y) = (x - y, 2x + y, -x).$$

Show that T is a linear transformation.

(b) Let $W : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear transformation such that $W\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $W\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$. Find $W\left(\begin{bmatrix} 1 \\ 7 \end{bmatrix}\right)$. (Hint: first write $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$.)

2. Consider the following system of linear equations

$$(1) \quad \begin{aligned} -8x_1 + 4x_2 + 2x_3 &= 2 \\ 4x_1 - 2x_2 &= 0. \end{aligned}$$

(a) Find ONE particular solution to the system (1).

(b) Find the general solutions to the homogeneous system

$$\begin{aligned} -8x_1 + 4x_2 + 2x_3 &= 0 \\ 4x_1 - 2x_2 &= 0. \end{aligned}$$

(c) Find the general solutions to the non-homogeneous system (1).

3. Let $S : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation given by

$$S(x_1, x_2) = (3x_1 + 5x_2, -x_1 - 2x_2).$$

(a) Find all \vec{x} such that $S(\vec{x}) = \vec{0}$.

(b) Determine whether S is one-to-one. Justify your answer.

(c) Determine whether S is onto. Justify your answer.

4. Let

$$A = \begin{bmatrix} 1 & -2 & -2 & -1 \\ -2 & 5 & 5 & 3 \\ 0 & 1 & 1 & 1 \end{bmatrix}.$$

(a) Are the columns of A linearly independent? Justify your answer.

(b) Do the columns of A span \mathbb{R}^3 ? Justify your answer.

5. Use elementary row operations to find the inverse A^{-1} of the following invertible matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}.$$

(Show all your steps.)