MATH106A,B CALCULUS II - PROF. P. WONG

EXAM I - FEBRUARY 6, 2009

NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	**Maximum Score**	**Your Score**
1.	20	
2.	20	
3.	20	
4.	20	
5.	20	
Total	100	

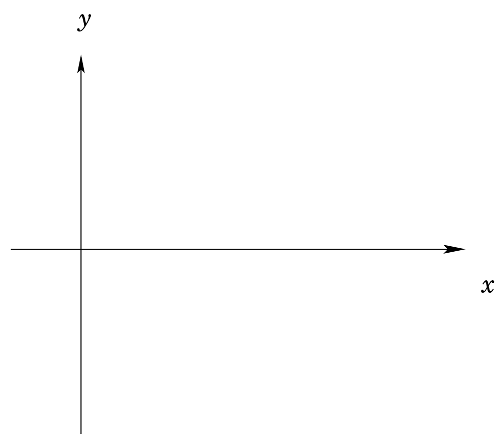
1.(10 pts.)(a) Find the **exact value** (by the Fundamental Theorem of Calculus) of the definite integral

$$\int_1^e \frac{\ln(2x)}{x}\, dx.$$

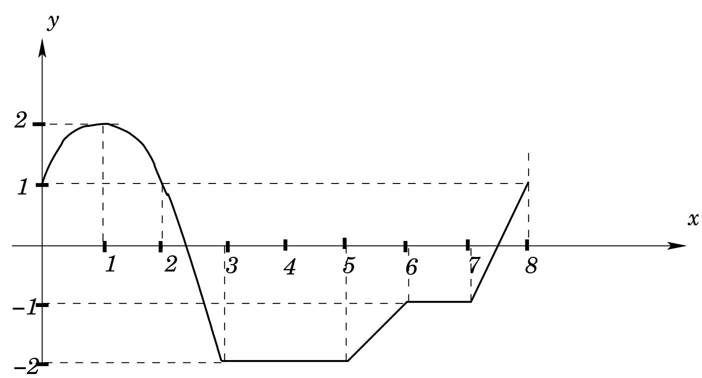
(10 pts.)(b) Evaluate the indefinite integral

$$\int xe^{(4-x^2)}\, dx.$$

2.(20 pts.) Find the area of the region bounded by the curve $x = 3y^2$ and the curve $x = y^2 + 2$. [Hint: sketch a picture of the region by determining the points of intersection between the curves.]



3. (10 pts.)(a) Consider a function h whose graph is given below.



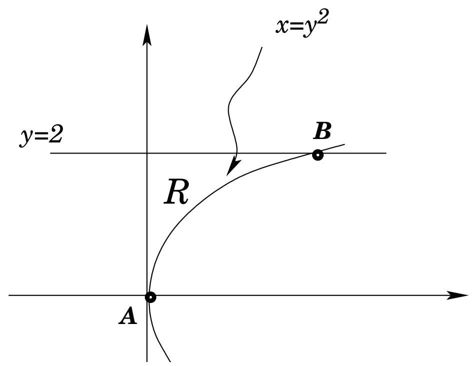
Find R_8, M_4 using the right-hand sum and the mid-point rule respectively for estimating the definite integral $\int_0^8 h(x)\ dx$.

(10 pts.)(b) Recall that the error committed by using the left hand sum approximation L_n is less than or equal to $\frac{K_1 \cdot (b-a)^2}{2n}$ where $|f'(x)| \leq K_1$ for some constant K_1 over the interval $[a, b]$. Use this result to give an upper bound for the error committed by L_{10} for

$$I = \int_0^2 (\cos x) e^x\ dx.$$

4. Let R be the region bounded by the curve $x = y^2$, the line $y = 2$, and the y-axis.

(15 pts.) Find the **exact volume** of the solid obtained from rotating the region R around the line $x = -1$.



(5 pts.) SET UP (**do not evaluate**) a definite integral representing the arc length of the portion of the curve $x = y^2$ from A to B [Hint: What is $f(x)$ here?].

5. Consider the initial value problem

$$\frac{dy}{dx} = y + xy$$

with $y(0) = 1$.

(10 pts.)(a) Use Euler's method to estimate the value $y(2)$ (when $x = 2$) of the solution using two steps with initial point $(0, 1)$. DO THIS BY HAND and show all your steps.

(10 pts.)(b) Use the technique of separation of variables to solve the Initial Value Problem.