

1. Suppose $T : \mathbf{R}^p \rightarrow \mathbf{R}^q$ is the linear transformation defined by $T \left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} 15x_1 + 10x_2 + 35x_3 \\ 16x_1 + 14x_2 + 24x_3 \\ 12x_1 + 10x_2 + 20x_3 \\ 10x_1 + 8x_2 + 18x_3 \end{bmatrix}$.

1a. What is the value of p ? ... and the value of q ?

1b. Find the image under T of $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

1c. Find the associated, or standard, matrix A of T .

1d. Use A in a supraugmented matrix to find explicit conditions on $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$ that will guarantee that

\mathbf{b} is in the range of T .

1e. Use (1d) to find a value for each of b_1 and b_2 so that $\mathbf{d} = \begin{bmatrix} b_1 \\ b_2 \\ 1 \\ 2 \end{bmatrix}$ is the image of some vector \mathbf{x} in \mathbf{R}^p .

1f. Let \mathbf{d} be as in the previous problem, and find at least one \mathbf{x} for which $T(\mathbf{x}) = \mathbf{d}$.

1g. Is T onto \mathbf{R}^q ? Explain in terms of the definition or any equivalent statement about A .

1g. Is T one-to-one? Explain in terms of the definition or any equivalent statement about A .

2. Now let $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be defined by $T \left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} 2x_1 + 3x_2 + 7 \\ x_1x_2 \end{bmatrix}$.

2A. Find the following:

$$T(\mathbf{e}_1) =$$

$$T(\mathbf{e}_2) =$$

$$T(\mathbf{e}_1 + \mathbf{e}_2) =$$

2B. Use the results from (2A) to explain why T is *not* a linear transformation.