

MATH 205A - Linear Algebra

EXAM 1 - 2/6/2004

PRINT your name

problem

1
2
3
4
total

- ① read the questions carefully,
- ② BE NEAT!
- ③ Show your work in the provided spaces.

Good Luck

FYI:

$$\begin{bmatrix} 1 & 5 & -6 & 14 & 5 & -34 & 15 & -5 \\ 2 & 11 & -14 & 29 & 3 & -80 & 30 & -12 \\ 3 & 17 & -17 & 49 & 1 & -101 & 55 & -14 \end{bmatrix} \text{ is row equivalent to}$$

$$\begin{bmatrix} 1 & 0 & 0 & 5 & 40 & 6 & 7 & 1 \\ 0 & 1 & 0 & 3 & -7 & -2 & 4 & 0 \\ 0 & 0 & 1 & 1 & 0 & 5 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 6 & 6 & 30 & -17 & 54 & 83 & 3 \\ 6 & 19 & 38 & 34 & 172 & -92 & 313 & 488 & 14 \\ 2 & 6 & 12 & 11 & 56 & -30 & 100 & 157 & 5 \end{bmatrix} \text{ is row equivalent to}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 6 & 1 & -9 & 5 & 3 \\ 0 & 1 & 2 & 0 & 0 & 2 & 5 & 8 & -2 \\ 0 & 0 & 0 & 1 & 4 & -4 & 8 & 9 & 1 \end{bmatrix}$$

Note: For clarity, questions 2c, 3b, 3d as they appear on the original class-given-version have been slightly edited in this version.

1. Consider the Linear Transformation T from \mathbf{R}^3 to \mathbf{R}^3 defined by $T(\mathbf{u}) = A\mathbf{u}$ where

$$A = \begin{bmatrix} 1 & 5 & -6 \\ 2 & 11 & -14 \\ 3 & 17 & -17 \end{bmatrix}.$$

- 1a. Find $T(\mathbf{v})$ where $\mathbf{v} = \begin{bmatrix} 5 \\ 3 \\ 1 \end{bmatrix}$.

- 1b. Find all of the solutions to $T(\mathbf{x}) = \mathbf{0}$. Explain how you know that you have the correct number of solutions.

- 1c. Determine if $\mathbf{b} = \begin{bmatrix} 15 \\ 30 \\ 55 \end{bmatrix}$ is in the image (or, range, as the book would say) of T .

Indeed, find *all* the vectors \mathbf{x} which T maps to \mathbf{b} .

2. Consider the system of equations corresponding to the augmented matrix

$$\left[\begin{array}{cccc|c} 1 & 3 & 6 & 6 & 83 \\ 6 & 19 & 38 & 34 & 488 \\ 2 & 6 & 12 & 11 & 157 \end{array} \right].$$

Let A be the matrix of coefficients of the system.

2a. Find all solutions of this system, and express them in the form $\mathbf{v}_h + \mathbf{p}$, as we have done in class.

2b. Find all the solutions of the corresponding homogeneous system of equations.

2c. Use your answer to (2b) to find a specific linear combination of the column vectors of A which “adds to” the zero vector in which *none* of the “weights” are 0, *or* explain why such a LC can’t be found.

2d. Are the column vectors of A linearly independent? Explain your answer in terms of the definition of linear independence.

2e. Which, if any, of the columns of A can be expressed as a LC of the other columns?

2f. Is the vector $\mathbf{v} = \begin{bmatrix} 30 \\ 172 \\ 56 \end{bmatrix}$ in the span of the column vectors of A ? If so, give an explicit LC of the columns that shows why.

3a. Put the following augmented matrix in reduced row echelon form (RREF). You may check with me to see if you got it right during the exam, without penalty if you made arithmetic errors.

$$\left[\begin{array}{cccc|c} 2 & 4 & 7 & 22 & a \\ 1 & 2 & 5 & 14 & b \\ 2 & 4 & 11 & 30 & c \end{array} \right].$$

3b. Write out the system of equations which corresponds to the augmented matrix in (3a).

3c. Does this system have solutions for *any* combination of a, b , and c ? If so, explain. If not, give any relationship(s) that a, b , and c must satisfy in order for a solution to exist.

3d. Give a specific example of a non-zero vector \mathbf{b} for which the system in (3a) has a solution, and proceed to find all solutions in the form $\mathbf{v}_h + \mathbf{p}$ where \mathbf{v}_h represents all solutions of the corresponding homogeneous equation and \mathbf{p} is a particular solution of (3a).

4. Suppose that T is a linear transformation from \mathbf{R}^2 to \mathbf{R}^2 . **4a.** What is that “rule” T must obey, when given any pair \mathbf{u} and \mathbf{v} of vectors in \mathbf{R}^2 ? Hint $T(\mathbf{u} + \mathbf{v})$ is involved.

4b. And what is that other rule T must obey, when given any vector \mathbf{w} in \mathbf{R}^2 and scalar α in \mathbf{R} ?

4c. Now, suppose you need to find $T\left(\begin{bmatrix} 44 \\ 13 \end{bmatrix}\right)$, and you know

$$T\left(\begin{bmatrix} 7 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 10 \\ 20 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 100 \\ 200 \end{bmatrix}.$$

Expressing $\begin{bmatrix} 44 \\ 13 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 7 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and using the fact that T obeys the rules for a linear transformation, you should be able to compute $T\left(\begin{bmatrix} 44 \\ 13 \end{bmatrix}\right)$. Do it.