MATH 205 EXAM 1 FEBRUARY 5, 2008

NAME: ______________________________

YOUR GRADE IS BASED ON CORRECTNESS, COMPLETENESS, AND CLARITY ON EACH EXERCISE. YOU MAY USE A CALCULATOR, BUT NO NOTES, BOOKS, OR OTHER STUDENTS. GOOD LUCK!

1.) (15 pts.) Consider the system of equations

$$\begin{array}{rcrcrcrcr} 6x_1 & - & 2x_2 & & & + & 13x_4 & = & 52 \\ -5x_1 & + & x_2 & + & 5x_3 & & & = & 12 \\ & & 8x_2 & - & x_3 & - & 11x_4 & = & -7. \end{array}$$

a.) (5 pts.) Write the system as a vector equation.

b.) (5 pts.) Write the system as a matrix equation.

c.) (5 pts.) Write the coefficient matrix of the system.

2.) (15 pts.) Given matrices $A = \begin{bmatrix} 7 & 35 & -28 \\ 3 & 16 & -9 \\ -2 & -12 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 & -4 \\ 3 & 16 & -9 \\ -2 & -12 & 3 \end{bmatrix}$, complete the following.

a.) (3 pts.) State which elementary row operation transforms A into B.

b.) (7 pts.) *BY HAND*, perform the row operations that transform B into reduced echelon form. (Continue onto the back of this page if you need more space.)

c.) (5 pts.) Is the system $A\mathbf{x} = \mathbf{b}$ consistent for every vector $\mathbf{b}$ in $\mathbb{R}^3$? Explain.

3.) (15 pts.)

a.) (5 pts.) Construct a 4×4 matrix, not in echelon form, whose columns span $\mathbb{R}^4$. Explain how you know the columns span $\mathbb{R}^4$.

b.) (5 pts.) Suppose a system of linear equations has a 2×4 augmented matrix whose fourth column is a pivot column. Is the system consistent? Why or why not?

c.) (5 pts.) Must a homogeneous system of equations always be consistent? Why or why not?

4.) (15 pts.)

a.) (5 pts.) What is a *quick* (meaning: no math needed) way to tell that the vectors $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} 41 \\ 13 \end{bmatrix}$, and $\begin{bmatrix} 23 \\ 2 \end{bmatrix}$ are linearly dependent?

b.) (5 pts.) Show that $\begin{bmatrix} 0 \\ 14 \\ 4 \end{bmatrix}$ is in $\text{Span}\left\{ \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} \right\}$.

c.) (5 pts.) How many pivot columns must a 6×3 matrix have if its columns are linearly independent? Why?

5.) (15 pts.)

a.) (5 pts.) Determine whether the system below has a nontrivial solution.

$$\begin{array}{rcrcrcl} x_1 & + & 3x_2 & - & 5x_3 & = & 0 \\ x_1 & + & 4x_2 & - & 8x_3 & = & 0 \\ -3x_1 & - & 7x_2 & + & 9x_3 & = & 0 \end{array}$$

b.) (5 pts.) Suppose the solution set of a certain system of linear equations can be described as $x_1 = 3x_4$, $x_2 = 8 + x_4$, $x_3 = 2 - 5x_4$, and x_4 free. Use parametric vector form to describe this set as a "line" in $\mathbb{R}^4$.

c.) (5 pts.) Let $T(\mathbf{e_1}) = \begin{bmatrix} 4 \\ -3 \\ -1 \end{bmatrix}$ and $T(\mathbf{e_2}) = \begin{bmatrix} -6 \\ -2 \\ 7 \end{bmatrix}$. Compute $T(2, -1)$.

6.) (15 pts.) For the parts below, consider $\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -6x_1 + 4x_2 + x_3 \\ 5x_1 - 8x_2 + 2x_3 \end{bmatrix}$.

a.) (5 pts.) Fill in the missing entries of the matrix, assuming that the equation holds for all values of the variables. Explain how you know what each entry should be.

b.) (5 pts.) Let A be the matrix you found in part (a). Is the transformation $T(\mathbf{x}) = A\mathbf{x}$ one-to-one? Explain your reasoning.

c.) (5 pts.) Let A again be the matrix you found in part (a). Is the transformation $T(\mathbf{x}) = A\mathbf{x}$ onto? Explain your reasoning.

7.) (10 pts.) A dietician is planning a meal that supplies certain quantities of vitamin C, calcium, and magnesium. Three foods will be used, their quantities measured in appropriate units. The nutrients supplied by these foods and the dietary requirements are given here.

	Milligrams (mg) of Nutrients per Unit of Food			
Nutrient	Food 1	Food 2	Food 3	Total Nutrients Required (mg)
Vitamin C	10	20	20	100
Calcium	50	40	10	300
Magnesium	30	10	40	200

Write a vector equation for this problem. State what the variables represent, and then solve the equations.