Math 106: Review for Exam I

1. Find the following. [Substitution tip: usually let u = a function that's "inside" another function, especially if du (possibly off by a multiplying constant) is also present in the integrand.]

   (a) $\displaystyle\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

   (b) $\displaystyle\int_\pi^{2\pi} \cos^7(5x)\sin(5x)\,dx$

   (c) $\displaystyle\int \frac{7x^2}{1+x^6}\,dx$

   (d) $\displaystyle\int_6^{10} x\sqrt{10-x}\,dx$

2. Suppose $f(x)$ is decreasing and concave up.

   (a) Put the following quantities in ascending order.

   L_{100}, R_{100}, T_{100}, M_{100}, $\int_a^b f(x)\,dx$

   (b) What can you say with certainty about where S_{200} would fit into your list above?

3. Suppose $f(t)$ is the rate of change (in animals per month) of a population $P(t)$.

   (a) What does $\int_4^{12} f(t)\,dt$ represent in this problem?

   (b) Find the best possible left, right, midpoint, trapezoidal, and Simpson's approximations to $\int_4^{12} f(t)\,dt$ given the data in the table below.

t	4	6	8	10	12
$f(t)$	15	11	8	4	3

4. Find bounds for each of the following errors if $I = \int_2^7 \ln x\,dx$.

   (a) $|I - L_{100}|$

   (b) $|I - T_{100}|$

   (c) $|I - M_{100}|$

5. If $I = \int_2^7 \ln x \, dx$, how many subdivisions are required to obtain a trapezoidal sum approximation with error of at most 1/1,000,000?

6. Use Euler's method with three steps on the differential equation $\frac{dy}{dt} = y - t$ to estimate $y(2.5)$ if $y(1) = 0$.

7. Solve the differential equation $dy/dx = 2xy + 6x$ if the solution passes through $(0, 5)$.

8. Write integrals equal to

   (a) the arc length of $y = x^2$ on the interval $[1, 5]$

   (b) the area bounded by $y = x^2 - 8x + 24$ and $y = 3x$

9. Consider the region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 9$. Write an integral equal to the volume generated if this region is rotated about

   (a) the x-axis

   (b) the line $x = -1$