NAME:

Instruction: Read each question carefully. Explain ALL your work and give reasons to support your answers.

Advice: DON’T spend too much time on a single problem.

<table>
<thead>
<tr>
<th>Problems</th>
<th>Maximum Score</th>
<th>Your Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>20</td>
<td></td>
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<tr>
<td>3.</td>
<td>20</td>
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<td>4.</td>
<td>20</td>
<td></td>
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<tr>
<td>5.</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
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1. (10 pts.) (a) Evaluate the indefinite integral
\[ \int \frac{\sin\left(\frac{1}{t}\right)}{t^2} \, dt. \]

(10 pts.) (b) Evaluate the indefinite integral
\[ \int x \cos(2x) \, dx. \]
2. (10 pts.) (a) Compute the \textbf{exact} value of the definite integral 
\[ \int_{2}^{3} \frac{1}{x^2 - 4x + 5} \, dx. \]

(10 pts.) (b) Consider a function \( f \) given by the following table.

\[
\begin{array}{c|cccc}
 x & 0 & 0.5 & 1 & 1.5 & 2 \\
 f(x) & 2 & 3 & 1 & 0 & -4 \\
\end{array}
\]

Find \( \text{TRAP}(4), \text{MID}(2) \) for the definite integral \( \int_{0}^{2} f(x) \, dx. \)
3. (15 pts.) (a) The region $R$ is bounded by the graph of $y = x^3$, the line $x = 1$ and the axis $y = 0$. Find the exact volume of the solid formed by revolving the region $R$ about the $x$-axis.

(5 pts.) (b) Write (DO NOT evaluate) a definite integral representing the arc-length of the path given by $y = x^3$ from the origin (0,0) to the point (1,1).
4. (7 pts.) (a) Compute the exact value of the improper integral, if it exists.

\[ \int_0^2 \frac{s}{\sqrt{4-s^2}} \, ds \]

(7 pts.) (b) Compute the exact value of the improper integral, if it exists.

\[ \int_0^\infty 3re^{-r^2} \, dr \]

(6 pts.) (c) Determine, by comparison, whether the following improper integral converges or not. Explain briefly.

\[ \int_1^\infty e^{-x^2} \, dx \]
5. While taking a walk along the road where you live, you accidentally drop your glove. You don’t know where you dropped it. Suppose the probability density $p(x)$ for having dropped the glove $x$ kilometers from home (along the road) is

$$p(x) = 2e^{-2x} \quad \text{for } x \geq 0.$$ 

(7 pts.) (a) What is the probability that you dropped it within 1 kilometer of home?

(7 pts.) (b) At what distance $y$ from home is the probability that you dropped it within $y$ km of home equal to 0.95?

(6pts.) (c) Where do you expect the glove to be found?