Show all your work to receive full credit for a problem.

1. (14 points) Evaluate the following integrals exactly (without using the table of integrals):
   (a) $\int_{1}^{2} x e^{x^2-1} \, dx$
   (b) $\int (15 - 2t) \ln t \, dt$

2. (21 points) Evaluate the following integrals. In case of an improper integral, find the value if the integral converges. Show clearly any limit computation you do. (If you use the table of integrals, mention the formula number and the values of the constants you use to find the answer.)
   (a) $\int (\cos \theta + \theta)^2 \, d\theta$
   (b) $\int_{3}^{4} \frac{7}{\sqrt{x-3}} \, dx$
   (c) $\int_{2}^{\infty} \frac{x+3}{x^2+6x+13} \, dx$

3. (5 points) Use comparisons to determine the convergence of the following integral:
   $$\int_{1}^{\infty} \frac{2x^3 + 29}{\sqrt{x^8 - 1}} \, dx$$

4. (4 points) The graphs of two functions $f(x)$ and $g(x)$ are given below:

   ![Graph of f(x) and g(x)]

   $g(x)$ is always below $f(x)$
Use these graphs to answer the following two questions:

(a) If \( \int_{1}^{\infty} f(x) \, dx \) converges, what (if anything) can you say about the convergence of \( \int_{1}^{\infty} g(x) \, dx \)? Briefly explain your answer.

(b) If \( \int_{1}^{\infty} f(x) \, dx \) diverges, what (if anything) can you say about the convergence of \( \int_{1}^{\infty} g(x) \, dx \)? Briefly explain your answer.

5. (9 points)

(a) Write (but do not evaluate) an integral equal to the volume of a hemisphere (half a sphere) of radius 4 feet.

(b) Use TRAP(25) to estimate the value of the integral you wrote in part (a). Give the formula for TRAP(25) in terms of LEFT(25) and RIGHT(25).

6. (10 points) Consider the region bounded by \( y = x^2 \), the \( x \)-axis and the line \( x = 2 \). Write (but do not evaluate) an integral to find the volume of the solid obtained by rotating the region about

(a) the \( x \)-axis

(b) the line \( x = 3 \)

7. (12 points) Water flows into a storage tank at a rate of \( r(t) \) ft\(^3\)/min, where \( t \) is the number of minutes since the water starts to flow in. The table below gives data for \( t \) and \( r(t) \):

<table>
<thead>
<tr>
<th>( t )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r(t) )</td>
<td>30</td>
<td>33</td>
<td>34</td>
<td>36</td>
<td>37</td>
<td>39</td>
<td>42</td>
</tr>
</tbody>
</table>

(a) What does the integral \( \int_{1}^{7} r(t) \, dt \) represent?

(b) Use the table to find the approximations LEFT(3) and LEFT(6) for \( \int_{1}^{7} r(t) \, dt \).

(c) Are the answers in part (b) underestimates or overestimates? Briefly explain your answer. (Assume that \( r(t) \) is increasing over the interval \([1,7]\).)

(d) Use your answers in part (b) to estimate the actual value of the integral.