LINEAR ALGEBRA EXAM 1 February 4, 2004

Name: ______________________________

While the final answer is important, you earn points for all the work leading to that answer, as well as the answer itself. Show all your steps clearly so you will be eligible for the most partial credit. Good luck!

1.) Provide examples of each of the following.

a.) (5 pts.) A 4×3 matrix, not in echelon form

b.) (5 pts.) A 3×4 matrix in echelon form, but not reduced echelon form

c.) (5 pts.) A 4×4 matrix with exactly three pivots

d.) (5 pts.) I_4

2.) a.) (10 pts.) Compute the reduced echelon form of the matrix $A = \begin{bmatrix} 1 & -1 & 5 & -7 \\ 2 & 0 & 7 & 3 \\ -3 & -5 & -3 & 2 \end{bmatrix}$.

SHOW ALL YOUR STEPS. You may use a calculator to check your work, but a calculator answer alone earns no credit.

b.) (5 pts.) Suppose A is an augmented matrix associated with a linear system. Write out the associated system of linear equations.

c.) (5 pts.) Is the system you wrote out in part (b.) consistent? Why or why not?

3.) Consider the vectors $\mathbf{u} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -4 \\ -1 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 0 \\ -11 \end{bmatrix}$.

a.) (10 pts.) Sketch $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, $2\mathbf{u}$, and $\mathbf{v} + \mathbf{w}$ on the same set of axes.

b.) (5 pts.) Describe, geometrically and in words, the space $\text{Span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$.

c.) (10 pts.) Are the vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ linearly independent? If yes, show why. If no, show how one is a linear combination of the other two.

4.) (10 pts.) Let A be a 3×2 matrix. Explain why the equation $A\mathbf{x} = \mathbf{b}$ cannot be consistent for all $\mathbf{b}$ in $\mathbb{R}^3$.

5.) (10 pts.) Show that the transformation T defined by $T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$ is not linear.

6.) a.) (5 pts.) **True or False:** If A and B are 2×2 with columns $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{b}_1$, $\mathbf{b}_2$, respectively, then $AB = \begin{bmatrix} \mathbf{a}_1\mathbf{b}_1 & \mathbf{a}_2\mathbf{b}_2 \end{bmatrix}$. If true, explain why. If false, correct the statement to make it true.

b.) (10 pts.) Compute AB if $A = \begin{bmatrix} 3 & 0 \\ -4 & -2 \\ -1 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 4 & 3 \\ -3 & -1 & 1 \end{bmatrix}$. NOTE: many of you can easily do this in your head, which is fine. To show work on this problem, write out all the details for ONE ENTRY of the matrix AB; it's not necessary to write out details for every entry of AB.