NAME:

Math 205 - Exam 1 - February 3, 2006

1. (8 pts.) Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form, where the matrix A is

$$\begin{bmatrix} 1 & -4 & -2 & 0 \\ 2 & -8 & 1 & 3 \\ 0 & 1 & -1 & 3 \end{bmatrix}.$$

   How can these solutions be described geometrically?

2. (8 pts. each) Consider the transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$, given by $T(\mathbf{x}) = A\mathbf{x}$, where A is row equivalent to

$$\begin{bmatrix} 1 & -4 & 8 & 1 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}.$$

   (a) Does T map $\mathbb{R}^4$ onto $\mathbb{R}^3$? Explain.

   (b) Is T one-to-one? Explain.

3. (8 pts.) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation that first reflect points through the vertical x_2-axis and then reflects points through the line $x_2 = x_1$. Find the standard matrix of T.

4. (4 pts. each) Consider the linear system

$$\begin{array}{rcrcl} x_1 & + & hx_2 & = & 2 \\ 3x_1 & + & 6x_2 & = & k \end{array}$$

For what value(s) of h and k (if any) does this system have

(a) no solutions?

(b) a unique solution?

(c) infinitely many solutions?

5. (6 pts. each) Let $A = \begin{bmatrix} 3 & 1 \\ -4 & -2 \end{bmatrix}$.

   (a) Calculate A^{-1}.

   (b) Using A^{-1}, find all solutions to the matrix equation $A\mathbf{x} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$, if any exist.

6. (3 pts. each) Answer the following as being true or false. You do **not** need to justify your answers.

   (a) If the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent, then $\mathbf{b}$ is not in the set spanned by the columns of A.

   (b) The columns of any 5×4 matrix are dependent.

   (c) Not every linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a matrix transformation.

   (d) For any $n \times n$ matrices A, B, and C, if $AB = AC$, then $B = C$.

7. (8 pts.) The *dot product* of two vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^n$, denoted by $\mathbf{u} \cdot \mathbf{v}$, is defined by $\mathbf{u} \cdot \mathbf{v} = \mathbf{u}^T \mathbf{v}$. For $\mathbf{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, find $\mathbf{v} \cdot \mathbf{v}$. How does this compare to the distance from the point $(0, 0)$ to the point $(3, 4)$? (Hint: Draw a right triangle and use the Pythagorean theorem..)

8. (8 pts.) Suppose A is a 3x4 matrix and $\mathbf{y}$ is a vector in $\mathbb{R}^3$ such that the equation $A\mathbf{x} = \mathbf{y}$ does *not* have a solution. Does there exist a vector $\mathbf{z}$ in R^3 such that the equation $A\mathbf{x} = \mathbf{z}$ has a unique solution? Explain your answer.

9. (8 pts.) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a linearly dependent set of vectors in $\mathbb{R}^n$. Explain why the set $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is linearly dependent.

10. (8 pts.) Suppose A is an invertible $n \times n$ matrix. Explain, without using the invertible matrix theorem, why the columns of A are linearly independent.

1	2	3	4	5	6	7	8	9	10	TOTAL