Math 106 Sections C and D
Test 1 (50 points)

Name: ________________________________

Show all your work to receive full credit for a problem.

There are seven questions. Questions are printed on both sides of a page.

You may use any of the following facts:

\[
\text{Arc length} = \int_a^b \sqrt{1 + (f'(x))^2} \, dx \quad \quad \quad \int u \, dv = uv - \int v \, du
\]

\[
|I - L_n| \leq \frac{K_1(b - a)^2}{2n} \quad \quad \quad \quad \quad |I - R_n| \leq \frac{K_1(b - a)^2}{2n}
\]

\[
|I - T_n| \leq \frac{K_2(b - a)^3}{12n^2} \quad \quad \quad \quad \quad |I - M_n| \leq \frac{K_2(b - a)^3}{24n^2}
\]

\[
|I - S_n| \leq \frac{K_4(b - a)^5}{180n^4} \quad \quad \quad S_{2n} = \frac{T_n + 2M_n}{3}
\]
1. (12 points) Evaluate the following integrals exactly (without using the table of integrals or the calculator integral function):

(a) \[ \int 2x \cos x \, dx \]

(b) \[ \int \frac{7x - 6}{3x - x^2} \, dx \]
2. (6 points) The quantity of water, \( Q(t) \), in a storage tank changes at a rate given by the following DE:

\[ \sqrt{1 + 5t^2} \, Q' = t. \]

The tank contains 10 cubic feet of water initially (at time \( t = 0 \)). Find the function \( Q(t) \) that gives the quantity of water in the tank after \( t \) minutes. (The quantity \( Q(t) \) is measured in cubic feet and the rate of change \( Q'(t) \) is measured in cubic feet per minute.) (Do not use the table of integrals or the calculator integral function.)
3. (7 points) Find the exact length of the curve $y = 3x^2$ from $x = 0$ to $x = 1$.

4. (6 points) Consider the region bounded by the curve $y = \ln x$, and the lines $x = 1$ and $x = e$ and the $x$-axis. Write (but do not evaluate) an integral to find the volume of the solid that is formed when the region is rotated about the line $y = 1$. 
5. (6 points) A tank in the shape of a right circular cone of height 18 ft and radius 4 ft is buried underground with its vertex pointing up and touching the ground level. If the tank is filled with gasoline (density = 42 lb/cubic foot) to a height of 14 ft, write (but do not evaluate) an integral equal to the work done in pumping all the gasoline in the tank to a height of 3 ft above ground level.

6. (6 points) Consider the IVP $y' = 2x + y$, $y(1) = 4$. Use Euler’s method with two steps to estimate $y(1.5)$. (Do not use a calculator program for this problem.)
7. (7 points) Let $f(x) = e^{-x^2}$ and let $I = \int_0^2 f(x) \, dx$.

(a) Use the error bound theorem to find a bound on the error committed by $L_{50}$. The graph of $f'(x)$ is given below for your convenience.

(b) Compute $L_{50}$ using your calculator program. (Round off your answer to four decimal places.) Using your answer in part (a), what can you say about the value of $I$?