

MATH106A CALCULUS II - PROF. P. WONG

EXAM I - FEBRUARY 2, 2007

NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	Maximum Score	Your Score
1.	20	
2.	20	
3.	20	
4.	20	
5.	20	
Total	100	

1.(10 pts.)(a) Evaluate the definite integral

$$\int_0^{\pi/2} e^{\sin x} \cos x \, dx.$$

(10 pts.)(b) Evaluate the indefinite integral

$$\int \frac{2x + 2x^3}{\sqrt{1 + x^2}} \, dx.$$

2.(20 pts.) Find the area of the region bounded by the curve $y = \ln x$, the x -axis, and the line $x = e(2 - y)$. [Hint: sketch a picture of the region by determining the points of intersections among these curves and lines]

3. (10 pts.)(a) Consider a function f given by the following table.

x	2	2.5	3	3.5	4
$f(x)$	2	0	4	-2	1

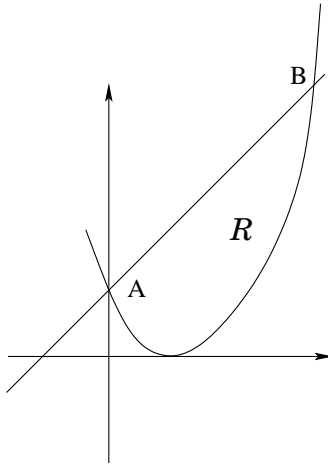
Find T_4 , M_2 using the trapezoidal and the mid-point rules respectively for the definite integral $\int_2^4 f(x) dx$.

(10 pts.)(b) Recall that the error committed by using the trapezoid approximation T_n is less than or equal to $\frac{K_2 \cdot (b-a)^3}{12n^2}$ where $|f''(x)| \leq K_2$ for some constant K_2 over the interval $[a, b]$. Use this result to give an upper bound for the error committed by T_8 for

$$I = \int_0^1 e^{x^2} dx.$$

4. Let R be the region bounded by the graph of $y = (x - 1)^2$ and the line $y = x + 1$.

(15 pts.) Find the volume of the solid obtained from rotating the region around the x -axis.



(5 pts.) SET UP (**do not evaluate**) a definite integral representing the arc length of the portion of the curve $y = (x - 1)^2$ from A to B where A and B are the points of intersection.

5. (10 pts.)(a) Consider the initial value problem

$$\frac{dy}{dx} = xy$$

with $y(0) = -2$.

Estimate the value $y(1)$ (when $x = 1$) of the solution using Euler's method with two steps with initial point $(0, -2)$. DO THIS BY HAND and show all your steps.

- (10 pts.)(b) SET UP (**do not evaluate**) a definite integral representing the work done in pumping fluid from the cone-shaped tank in the figure to the rim. The fluid has density ρ (constant) and the depth of the fluid is 10m. [gravitational constant is g]

