

NAME:

Math 106B - Exam 1 - February 2, 2007

INSTRUCTIONS: Show all of your work and circle your solutions. Cross out any unnecessary work.

1. (10 points each)

(a) $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

(b) $\int x(2x + 1)^9 dx$

2. Let $I = \int_1^7 v(t) dt$, where particular values of the function $v(t)$ are given in the table below. (Note the endpoints of the integral!)

t	0	1	2	3	4	5	6	7	8
$v(t)$	6.3	4.4	2.9	1.6	0.6	-0.3	-0.9	-1.4	-1.7

- (a) (4 points) Suppose $v(t)$ gives the eastward velocity at time t (seconds), in feet per second, of a runner. What does I (defined above) represent?
- (b) (4 points) Approximate I with R_2 , or explain why you cannot evaluate it with the information given.
- (c) (4 points) Approximate I with M_3 , or explain why you cannot evaluate it with the information given.
- (d) (3 points) $v(t)$ appears to be decreasing and concave up. Assuming that it is, will T_6 be an underestimate for I , an overestimate for I , or can we not tell? (No explanation necessary.)

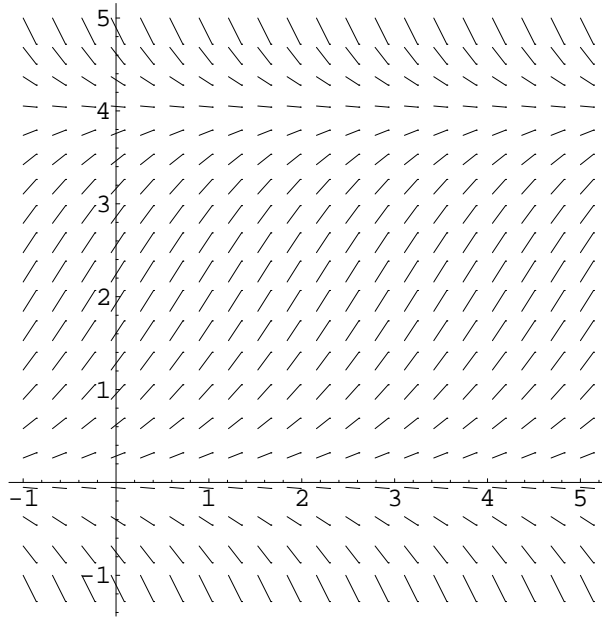


Figure 1: $y' = .4y(4 - y)$

3. (5 points) Consider the differential equation $y' = .4y(4 - y)$. The slope field is given above.
- Sketch the solution for which $y(2) = 1$. (Be sure to sketch the solution in both directions, i.e., to the left and right of $t = 2$.) Label the point on your sketch where $t = 4$.
 - From your solution sketch, estimate the value of $y(4)$.

4. (10 points) Consider the initial value problem

$$\begin{cases} y' = 2ty \\ y(1) = -1 \end{cases}$$

(This has nothing to do with the previous problem or its slope field!) Use Euler's method with two steps to estimate $y(3)$.

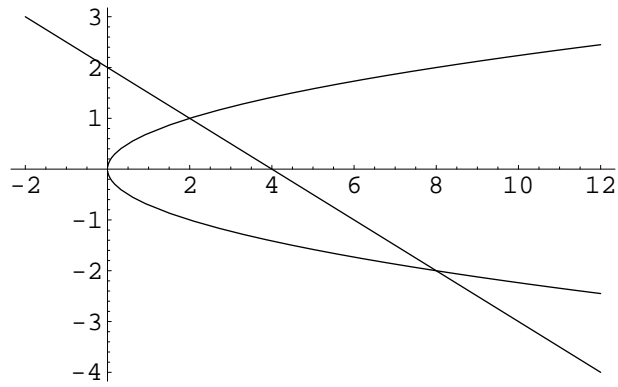


Figure 2: Graphs of $x = 2y^2$ and $y = -\frac{x}{2} + 2$

5. (10 points each) Consider the region in the xy -plane bounded by the graphs $x = 2y^2$ and $y = -\frac{x}{2} + 2$. In each part below, set up, but don't evaluate, an integral, or if necessary, a sum of two integrals, that represents the quantity given below.

(a) The area of the region, where the integral(s) is (are) done with dx .

(b) The area of the region, where the integral(s) is (are) done with dy .

(Problem 5 continued. Set up the integral(s) to calculate the quantity. You do not need to evaluate the integral(s).)

(c) The volume of the solid obtained by rotating the region around the line $x = 10$.

6. (10 points) Let $I = \int_2^4 x \ln x \, dx$. Approximating I with a midpoint sum M_n , how large must n be to guarantee accuracy to within $\frac{1}{1000}$ of I ?

7. (10 points) A 40 pound bucket, at the bottom of a 70 foot well, is connected to the top by rope that weighs 2 pounds per foot. How much work is required to raise the bucket (and rope) to the top of the well? (Set up the integral - you do not need to evaluate it.)

1	2	3	4	5	6	7	TOTAL