MATH 106, Section D

Test # 1

1. (20 points) Evaluate the integral.

$$\int \frac{\cos(\ln x)}{x} dx$$

2. (20 points) Find the minimal value of n such that R_n approximates the value of the integral with the absolute error less than or equal to 0.007.

$$\int_1^e x \ln x dx$$

HINT: Let $I = \int_a^b f(x)\, dx$ and K_1 be the minimal positive number such that $|f'(x)| \leq K_1$ on $[a, b]$, then

$$|I - R_n| \leq \frac{K_1 (b-a)^2}{2n}.$$

3. (20 points) Find the area of the region in the xy-plane bounded by the curves $y = e^{2x}$, $y = e^x$, and $x = \ln 2$.

4. (20 points) Find the length of the curve. $y = \ln x - \frac{x^2}{8}$, $x \in [1, 4]$

5. (20 points) Find the volume of the solid of revolution formed when the region bounded by $y = 1 - \sqrt{x}$, $y = x + 1$, and $x = 1$ is revolved about the line $y = 3$.

6. (20 points) A cylindrical gasoline tank with vertical axis of symmetry, radius 20 feet, and height 60 feet is located on the ground. One third of the tank is empty. Gasoline in the tank weighs 43 lb/ft^3. Find the amount of work needed to pump all the gasoline in the tank to a nozzle that is 15 feet above the top of the tank.