Math 206 — First Midterm

February 1, 2012

Name: ______________________________

Instructor: ______________________ Section: __________

1. **Do not open this exam until you are told to do so.**
2. This exam has 7 pages including this cover AND IS DOUBLE SIDED. There are 8 problems. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
3. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out when you hand in the exam.
4. Please read the instructions for each individual problem carefully. One of the skills being tested on this exam is your ability to interpret mathematical questions.
5. Show an appropriate amount of work (including appropriate explanation). Include units in your answer where that is appropriate. Time is of course a consideration, but do not provide no work except when specified.
6. You may use any previously permitted calculator. However, you must state when you use it.
7. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph that you use.
8. **Turn off all cell phones and pagers**, and remove all headphones and hats.
9. Remember that this is a chance to show what you've learned, and that the questions are just prompts.

Problem	Points	Score
1	15	
2	20	
3	14	
4	14	
5	12	
6	09	
7	14	
8	02	
Total	100	

1. [15 points] Match the following three curves and surfaces with the equations (there are three unused equations). No partial credit, no explanation needed (5 points for each correct match...).

1.$x^2 + 2x - y^2 = (z + 2)^2$

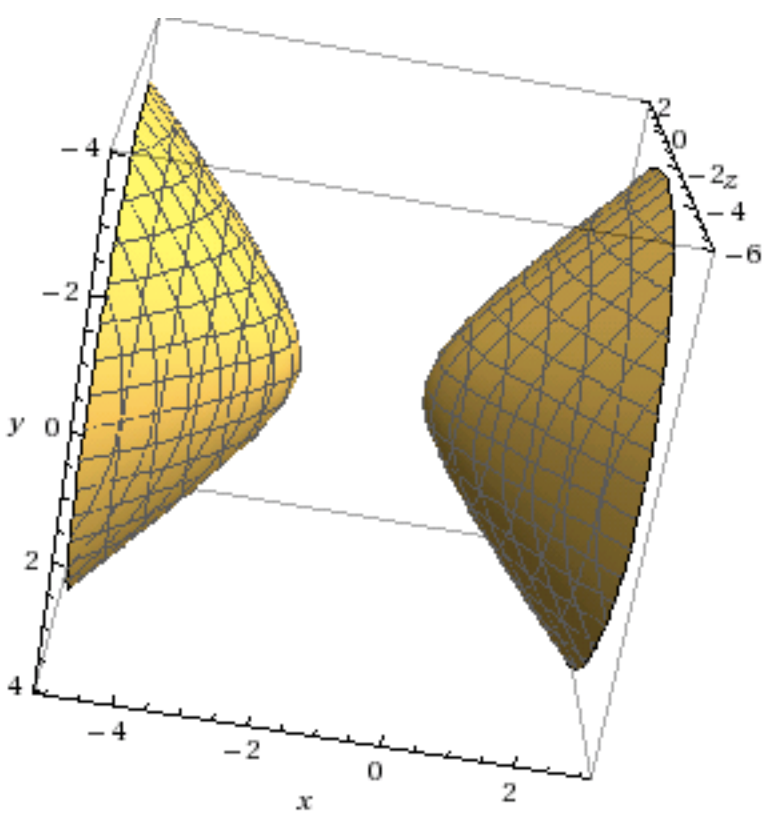
2.$[t(t-1), t^2(t-1), t(t-1)^2]$ for any real t

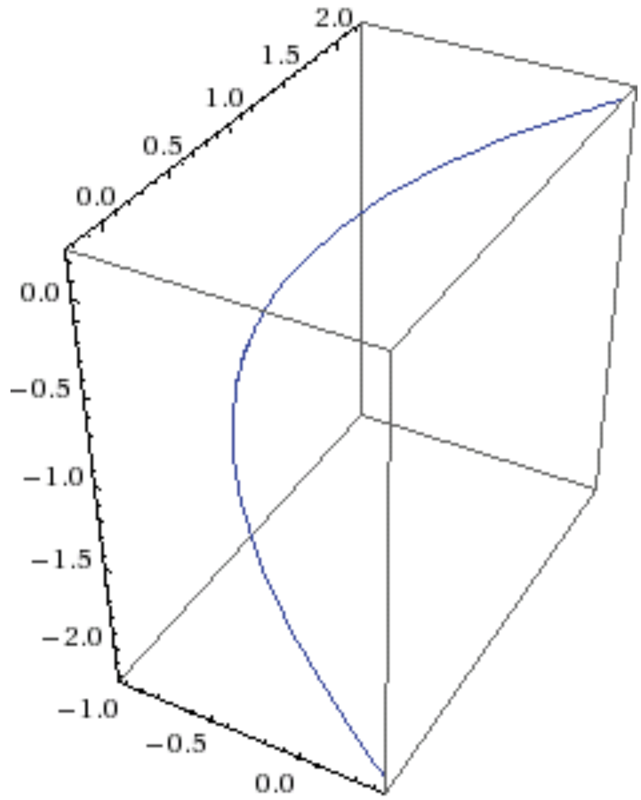
3.$[3\sin(2t), 2\cos(3t), t]$ for any real t

4.$x = e^{y^2+z^2}$

5.$z = e^{x^2+y^2}$

6.$[t^2 - 1, t + 1, t - 1]$ for any real t

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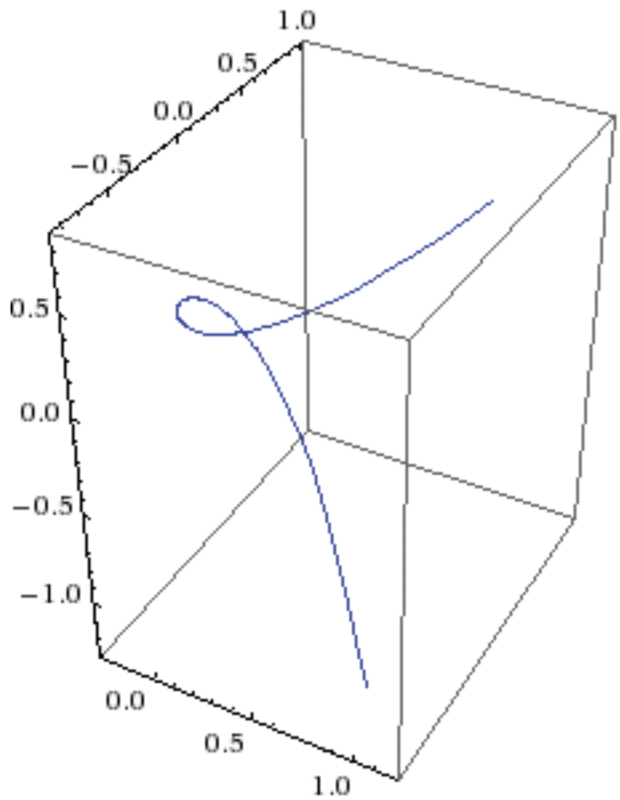
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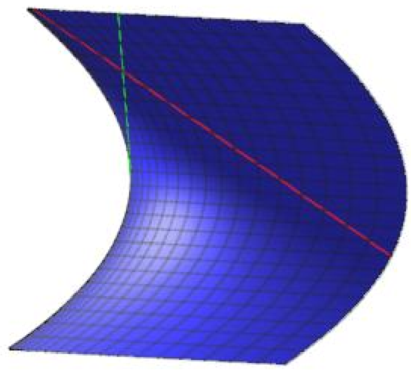
2. [20 points]
The surface described by $z = x^2 - y^2$ is the hyperbolic paraboloid (or "saddle") that we've seen in class (or in the image below). What you may not know, is that this is also the general shape used for cooling towers in power plants (see below). The reason is that even though this is a curved surface, there are straight lines for steel beams contained inside it (note the red and green lines). Let's explore:



a. [8 points] Check that the two functions below parametrize two straight lines which are contained in the surface $z = x^2 - y^2$? Briefly explain your work (including whether these ARE lines).

•$f(t) = [t + 1, t - 1, 4t]$ for any real t

•$g(s) = [s + 1, 1 - s, 4s]$ for any real t

b. [5 points] Find the point where these two lines intersect?

c. [7 points] Find the tangent plane of the hyperbolic paraboloid at that point.

3. [14 points] Calculate the following limits, or demonstrate that they do not exist:

a. [7 points]

$$f(x,y) = \frac{x^3 y}{x^6 + y^2}$$

What is $\lim_{(x,y)\to(0,0)} f(x,y)$ or does it not exist?

b. [7 points] Does

$$\lim_{t\to 0} \frac{(x+t)^2\cos(y) - x^2\cos(y)}{t}$$

exist? If so what is it?

4. [14 points] Explain why two level curves of a function $f(x, y)$ cannot intersect.

5. [12 points]

a. [6 points] Let $u = (-3, 2, 3)$. Find a vector v of length 1 so that $u + v$ is as long as possible.

b. [6 points] Let $u = (-3, 2, 3)$. Find a vector w of length 1 so that $|u \times w|$ is as small as possible.

6. [9 points] Sketch the image of the unit square ($1 \leq x \leq \mathrm{e}$ and $0 \leq y \leq 1$) under the function $f(x, y) = (\ln(x)y, x\,\mathrm{e}^y)$

7. [14 points]

a. [7 points] If $f(t) = (t^3 - t, t^2 + 1, t - 5)$ what is $f'(2)$?

b. [7 points] What is the tangent line at $f(2)$ to the curve parametrized in the first part of this problem?

8. [2 points] Who do you think will win the superbowl? The Ravens or the Forty-niners?