MATH106A,B CALCULUS II - PROF. P. WONG

EXAM I - FEBRUARY 1, 2008

NAME:

Instruction: Read each question carefully. Explain **ALL** your work and give reasons to support your answers.

Advice: DON'T spend too much time on a single problem.

Problems	**Maximum Score**	**Your Score**
1.	20	
2.	20	
3.	20	
4.	20	
5.	20	
Total	100	

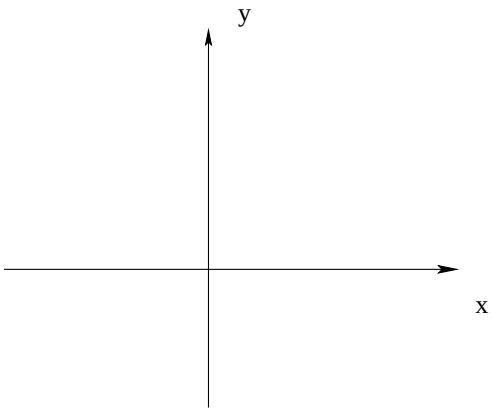
1.(10 pts.)(a) Find the **exact value** (by the Fundamental Theorem of Calculus) of the definite integral

$$\int_1^e \frac{1+(\ln x)^2}{x}\,dx.$$

(10 pts.)(b) Evaluate the indefinite integral

$$\int \frac{x}{\sqrt{1-x^4}}\,dx.$$

2.(20 pts.) Find the area of the region bounded by the curve $x = 4 - y^2$ and the line $2y = 1 - x$. [Hint: sketch a picture of the region by determining the points of intersections between the curve and the line]



3. (10 pts.)(a) Consider a function h given by the following table.

x	1	1.5	2	2.5	3	3.5	4
$h(x)$	-1	2	1	0	-2	3	1

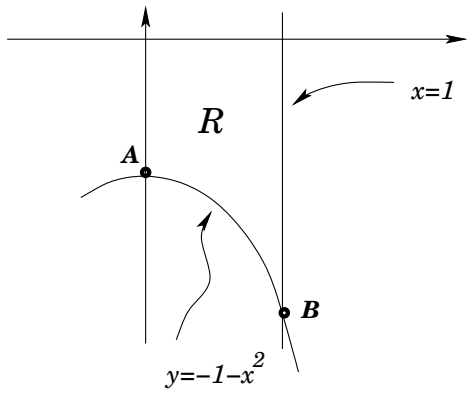
Find R_6, M_3 using the right-hand sum and the mid-point rule respectively for estimating the definite integral $\int_1^4 h(x)\ dx$.

(10 pts.)(b) Recall that the error committed by using the left hand sum approximation L_n is less than or equal to $\frac{K_1 \cdot (b-a)^2}{2n}$ where $|f'(x)| \leq K_1$ for some constant K_1 over the interval $[a, b]$. Use this result to give an upper bound for the error committed by L_{10} for

$$I = \int_0^2 (\sin x)e^x\ dx.$$

4. Let R be the region bounded by the curve $y = -1 - x^2$, the line $x = 1$, the x-axis and the y-axis.

(15 pts.) Find the **exact volume** of the solid obtained from rotating the region R around the x-axis.



(5 pts.) SET UP (**do not evaluate**) a definite integral representing the arc length of the portion of the curve from A to B.

5. (10 pts.)(a) Consider the initial value problem

$$\frac{dy}{dx} = \frac{x}{x+y}$$

with $y(1) = 1$.

Use Euler's method to estimate the value $y(2)$ (when $x = 2$) of the solution using two steps with initial point $(1, 1)$. DO THIS BY HAND and show all your steps.

(10 pts.)(b) Use the technique of separation of variables to solve the following Initial Value Problem

$$y' = \frac{y}{\sqrt{1-x^2}}, \qquad y(0) = 1.$$