

NAME _____

I _____ II _____ III _____ IV _____ V _____ VI _____ VII _____ VIII _____ TOTAL _____

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Mathematics 309a
Abstract Algebra
Examination #1

Mr. Haines

(10 points) I. Define any two of these terms. Use a complete, mathematically correct sentence for each definition.

binary operation on a set S
isomorphism from group $\langle G, * \rangle$ to group $\langle G', *' \rangle$
trivial subgroup of a group $\langle G, * \rangle$

A.

B.

(15) II. Give examples of:

A. a set with two elements and a binary operation on S that is not associative. Explain why your operation is not associative. Use this table to define your operation:

B. an abelian group with 49 elements.

C. a cyclic subgroup of $\langle \mathbb{R}^*, \cdot \rangle$, the non-zero real numbers under multiplication.

(10) III. Draw the lattice diagram of subgroups of Z_{125} , the integers modulo 125, where the operation is $+_{125}$.

(10) IV. Find all solutions in \mathbb{C} to the equation $z^8 - 1 = 0$.

(10) V. The identity function, id , is one of four automorphisms of Z_8 . If the others are denoted ϕ_1, ϕ_2 , and ϕ_3 , complete this table:

x	$id(x)$	$\phi_1(x)$	$\phi_2(x)$	$\phi_3(x)$
0				
1				
2				
3				
4				
5				
6				
7				

(20) VI. Let H be the subset $M_2(\mathbb{R})$ consisting of all matrices of the form $\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$ for $b \in \mathbb{R}$.

A. Show H is not closed under matrix addition, denoted $+$.

B. Show H is closed under matrix multiplication, denoted $*$.

C. Show that H contains the identity under matrix multiplication.

D. Show that H is a group under matrix multiplication.

E. Describe the cyclic subgroup K of H generated by $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

F. Give an isomorphism from $\langle \mathbb{Z}, + \rangle$ to $\langle K, * \rangle$.

(15) VII. If $\langle G, * \rangle$ is an abelian group with identity e , define $H = \{x \in G \mid x * x = e\}$.

A. If $\langle G, * \rangle = \langle \mathbb{Z}_{30}, +_{30} \rangle$, what is H ?

B. Prove that $H \leq G$, i. e. that H is a subgroup of G .

(10) VIII. TRUE OR FALSE? (Don't guess! The number of incorrect responses will be subtracted from the number of correct ones. Thus, random guessing earns you no points at all.)

_____ 1. The set of all real numbers is closed under subtraction.

_____ 2. Every abelian group has an identity.

_____ 3. Any two groups with 4 elements are isomorphic.

_____ 4. The empty set is an example of a group.

_____ 5. Every cyclic group has a generator.

_____ 6. All groups are cyclic.

_____ 7. The set of all real numbers form a group under addition.

_____ 8. The set of all real numbers form a group under multiplication.

_____ 9. Subtraction of integers is associative.

_____ 10. Every isomorphism of binary structures must be one-to-one and onto.