NAME:

Show ALL your work CAREFULLY.

(a) Decide whether the following improper integral converges or diverges. If it converges, find its value.
\[ \int_0^\infty x e^{-x} \, dx. \]

The improper integral is
\[ \int_0^\infty x e^{-x} \, dx = \lim_{b \to \infty} \int_0^b x e^{-x} \, dx \]
\[ = \lim_{b \to \infty} \left[ -x e^{-x} \bigg|_0^b - \int_0^b e^{-x} \, dx \right] \]
\[ = \lim_{b \to \infty} \left[ -be^{-b} + e^{-x} \bigg|_0^b \right] \]
\[ = \lim_{b \to \infty} \left[ -(be^{-b} + (e^{-b} - 1)) \right] \]
\[ = 1. \]

(b) Consider the region \( A \) bounded by the graph of \( y = \sqrt{x} \), the line \( x = 1 \) and the axis \( y = 0 \). Set up the definite integral representing the volume of revolution by rotating the region \( A \) about the line \( x = 1 \). Find the volume of this solid by evaluating the integral. (Sketch the region on the other side.)

See the figure on the back. A typical slice of the solid has volume \( \Delta V \approx \pi (1 - x)^2 \Delta y \). Since \( y = \sqrt{x} \), we have \( x = y^2 \). The volume of the solid of revolution is then given by
\[ \int_0^1 \pi (1 - y^2)^2 \, dy = \int_0^1 \pi \left(1 - 2y^2 + y^4\right) \, dy \]
\[ = \pi \left[y - \frac{2}{3}y^3 + \frac{1}{5}y^5\right]_0^1 \]
\[ = \pi \left( 1 - \frac{2}{3} + \frac{1}{5} \right) = \frac{8\pi}{15}. \]
2Q QUIZ 3

A

1-x

(1,1)

x=1

y

x

x