

1. Consider the system of equations

$$(1) \quad \begin{aligned} x_1 + 4x_2 + 16x_3 - 10x_4 &= p \\ 3x_1 + 10x_2 + 38x_3 - 24x_4 &= -17 \\ qx_1 + 12x_2 + 40x_3 - 26x_4 &= -19 \\ -2x_1 - 8x_2 - 32x_3 + 20x_4 &= 14 \end{aligned}$$

Here p and q are two numbers whose values you will be able to determine after you complete question 1A. Now, after writing it as an augmented matrix, the above system (1) is row equivalent to

$$\left[\begin{array}{cccc|c} 1 & 0 & -4 & 2 & 1 \\ 0 & 1 & 5 & -3 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right].$$

1A. What are the solutions of the system (1)? Write your answer in the form $\mathbf{x} = \mathbf{v}_h + \mathbf{p}$ where \mathbf{v}_h represents all solutions of the homogeneous system corresponding to the system (1), and \mathbf{p} is a particular solution of (1).

1B. Now you should be able to determine the values of p and q . What are they?

1C. Short answer: Can we replace the last column $[p, -17, -19, 14]$ in (1) with *any* column $[a, b, c, d]$ and be guaranteed that a solution will exist?

1D. Short answer: Do the columns of the matrix A of coefficients of the system in (1) span \mathbf{R}^4 ?

1E. Short answer: Are questions 1C and 1D asking the same thing?

NOTE WELL: In answering questions 2 and 3 on this page, “*this is impossible, no matter how a, b, c and d are chosen*” might be a legitimate answer. If so, write “IMPOSSIBLE” where it is appropriate.

2. Suppose when written as an augmented matrix, $A\mathbf{x} = \mathbf{b}$ is row equivalent to

$$\left[\begin{array}{cccc|c} 1 & 3 & 0 & 0 & a \\ 0 & 0 & 1 & 0 & b \\ 0 & 0 & 0 & 1 & c \\ 0 & 0 & 0 & 0 & d \end{array} \right].$$

What conditions must a, b, c and d satisfy in order for $A\mathbf{x} = \mathbf{b}$ to have...

2a. ...exactly one solution?

2b. ...no solutions?

2c. ...infinitely many solutions?

2d. Do the columns of A span \mathbf{R}^4 ?

2e. Find a non-trivial solution to $A\mathbf{x} = \mathbf{0}$.

2f. Are the columns of A linearly independent? Explain.

3. Suppose when written as an augmented matrix, $A\mathbf{x} = \mathbf{b}$ is row equivalent to

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & a \\ 0 & 1 & 2 & 3 & b \\ 0 & 0 & 1 & 2 & c \\ 0 & 0 & 0 & 1 & d \end{array} \right].$$

What conditions must a, b, c and d satisfy in order for $A\mathbf{x} = \mathbf{b}$ to have...

3a. ...exactly one solution?

3b. ...no solutions?

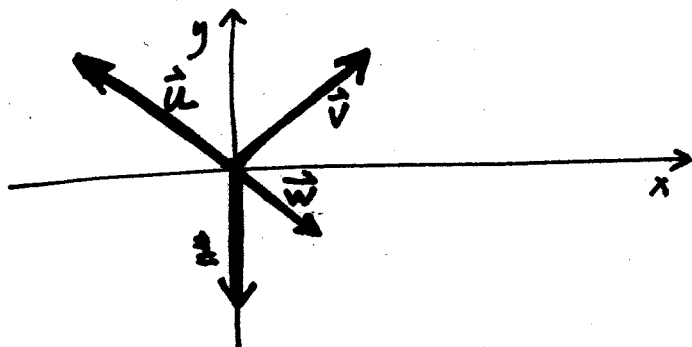
3c. ...infinitely many solutions?

3d. Do the columns of A span \mathbf{R}^4 ?

3e. Is there a non-trivial solution to $A\mathbf{x} = \mathbf{0}$?

3f. Are the columns of A linearly independent? Explain.

The following illustration is drawn in \mathbb{R}^2 , ie, all vectors belong to \mathbb{R}^2 . Use it to answer questions 4 and 5:



4 What is span of the vectors in each case? Answers might be phrased like “the line through...” or “all of \mathbb{R}^2 , etc.

- (A) span of $\{w\}$
- (B) span of $\{w, z\}$
- (C) span of $\{w, u\}$
- (D) span of $\{u, v, z\}$

5 Which of the following sets of vectors are linearly independent (L.I.)? Write “L.I.” next to those which are, and “not L.I.” otherwise.

- (A) $\{w\}$
- (B) $\{w, z\}$
- (C) $\{w, u\}$
- (D) $\{u, v, z\}$

6. What augmented matrix would have resulted in the following solution set?

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \mathbf{v}_h + \mathbf{p} = x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 3 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ 0 \\ -4 \\ 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 5 \\ 0 \\ -6 \\ 0 \\ 0 \\ 7 \end{bmatrix}$$