

Math 205 Section B
Test 1 (50 points)

Name: _____

- Check that you have 6 questions on two pages.
- Show all your work to receive full credit for a problem.

1. **(10 points)** (For this problem do all calculations by hand.) Determine h and k such that the solution set of the following system

(a) is empty

(b) contains a unique solution

$$\begin{aligned}x_1 + 5x_2 &= h \\2x_1 + kx_2 &= 0\end{aligned}$$

2. **(10 points)** (For this problem do all calculations by hand.) Let $A = \begin{bmatrix} 1 & 2 \\ -2 & -3 \\ 3 & 5 \end{bmatrix}$ and

$$\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

(a) Does the equation $A\vec{x} = \vec{b}$ have a solution for all possible \vec{b} ? Explain.

(b) Describe the set of all \vec{b} for which the equation $A\vec{x} = \vec{b}$ does have a solution.

3. (8 points) Let $\vec{v}_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 6 \\ -3 \\ 0 \end{bmatrix}$.

(a) List three vectors in $\text{Span}\{\vec{v}_1, \vec{v}_2\}$. For each vector, show the weights on \vec{v}_1 and \vec{v}_2 used to generate the vector.

(b) Give a geometric description of $\text{Span}\{\vec{v}_1, \vec{v}_2\}$.

4. (8 points) Let $A = \begin{bmatrix} 1 & -3 & -2 & 5 \\ -4 & 0 & -4 & -8 \end{bmatrix}$. Describe all solutions of $A\vec{x} = \vec{0}$ in parametric vector form.

5. **(8 points)** Let $A = [\vec{a}_1 \ \vec{a}_2 \ \vec{a}_3]$ be a 4×3 matrix. ($\vec{a}_1, \vec{a}_2, \vec{a}_3$ are vectors that form the columns of A .) Suppose $\begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ is a solution of the equation $A\vec{x} = \vec{0}$.

(a) Find numbers $x_1, x_2,$ and x_3 such that $x_1\vec{a}_1 + x_2\vec{a}_2 + x_3\vec{a}_3 = \vec{0}$.

(b) Are the columns of A linearly independent? Explain.

(c) Do the columns of A span \mathbb{R}^4 ? Explain.

6. **(6 points)** Let A be a 3×2 matrix. Let T be the linear transformation given by $T(\vec{x}) = A\vec{x}$. Answer the following questions for this A and T .

(a) What is the domain of T ? Circle only one choice: \mathbb{R}^2 \mathbb{R}^3

(b) Suppose $T(\vec{u}) = \begin{bmatrix} -7 \\ 3 \\ 4 \end{bmatrix}$ and $T(\vec{v}) = \begin{bmatrix} 2 \\ 0 \\ -11 \end{bmatrix}$. Find $T(\vec{u} - 2\vec{v})$.