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I_____II_____III_____IV_____V_____VI_____VII_____VIII_____ TOTAL _____

January 28 2005 — Mathematics 206a Multivariable Calculus Examination #1 — Mr. Haines

(10) I. Give the parametric equation of the line segment connecting the point (1, 2, 3) and the point (2, 5, 7) . This is a line segment of finite length, so be sure to put the proper limits on your parameter.

(10) II. Give a coordinate equation for the plane containing the point (1, 2, 5) which is perpendicular to the cross product of the vectors $\mathbf{v}_1 = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{v}_2 = \mathbf{i}$.

(20) III. A plane P in $\Re^3$ has equation $x - y = 0$.

A. Give a unit vector that is perpendicular to P.

B. Give a point that is in P.

C. Give the components of two non-parallel vectors which are perpendicular to the normal to P.

D. Give a parametrization of P.

(10) IV. Here are the four corners of a parallelogram in $\Re^3$:

(1, 1, 1), (2, 4, 3), (3, 2, 4), and (4, 5, 6).

What is its area?

(10) V. Calculate the integral:

$$\int \left(t^3 \mathbf{i} + (\sin 2t)\mathbf{j} + \left(\sqrt{\frac{1}{2t}} \right) \mathbf{k} \right) \mathbf{d}t$$

(10) VI. Give examples of:

A. A line in $\Re^4$.

B. An equation for any cone in $\Re^3$.

C. A negative definite quadratic form in three variables.

(20) VII. Suppose $\mathbf{A} = \begin{bmatrix} 6 & 4 \\ 2 & 3 \end{bmatrix}$ and $\mathbf{T} : \Re^2 \rightarrow \Re^2$ is a linear transformation with the formula $\mathbf{T}(\mathbf{x}) = \mathbf{Ax}$. Suppose $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j}$

A) What is $\mathbf{T}^{-1}(\mathbf{a})$?

B) What is $\mathbf{T}(\mathbf{T}^{-1}(\mathbf{a}))$?

(10)VIII. The **unit tangent vector** (also called the **unit velocity vector**) to the path determined by the vector-valued function **f** is the unit vector that is tangent to the path. For the path $\mathbf{f}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$,

A. give the unit tangent vector at the point where $t = 1$.

B. give the equation of the tangent line to this path at the point $\mathbf{f}(1)$.