1. The graph of a certain function \( h(x) \) is plotted below. On the axes beneath it, plot the graph of \( h'(x) \). Note that three important tangent lines are drawn in; use these to make good estimates of \( h'(x) \) at the corresponding \( x \)-coordinates.

There are 3 \( x \)-coords, where the slope on \( h(x) \) is 0, namely \( x = 0, 2 \) and 4.

We've written some slopes on this graph...

these give us additional \( y \)-coordinates below at the respective \( x \)-coords.

Note \( h(x) \) is not differentiable at \( x = 7 \).
2. The graph of the derivative $g'(x)$ of a certain function $g(x)$ is plotted below. Use the graph of $g'$ to answer questions about $g$. If it is impossible to answer a question, write "N/A".

a. On what intervals is $g'$ increasing? $(-2, 1.5)$ and $(5, 8)$

b. On what intervals is $g$ concave down? $g'$ is decreasing on $(1.5, 5)$

c. At what $x$ values strictly between $-2$ and $8$ does $g$ have a local maximum?
   - At $x=3.5$, since $g'(3.5)=0$ and $g$ is increasing on the left of $3.5$ and decreasing on the right (since $g'$ is positive and negative, respectively)

d. On what intervals is $g$ increasing?
   - $g'$ is positive, so $(-0.5, 3.5)$ and $(6.5, 8)$

e. At what $x$ values strictly between $-2$ and $8$ does $g'$ have a local maximum?
   - At $x=1.5$ (Note that a local max at $x=8$, but $8$ is not strictly between $-2$ and $8$)

f. On what intervals is $g$ positive?
   - There is NO WAY to tell

g. On what intervals is $g''$ negative?
   - $g''$ is negative when $g'$ is decreasing, $(-1.5, 5)$

h. What are the $x$ coordinates of the inflection points of $g$?
   - We need $g'$ to change from increasing to decreasing (or vice versa)
   - Hence at $x=1.5$ and $x=5$
3. Use our “rules” for finding derivatives to find the derivatives of each of the following. Show all your work and write your answers in terms of “radicals”; do not leave negative nor fractions in the exponents.

a. \( f(x) = 4x^5 + \sqrt{2}x^3 - 1/x^2 + 20 \)
\[
= 4x^5 + \sqrt{2}\ x^3 - x^{-2} + 20
\]
\[
f'(x) = 20x^{4 + \sqrt{2}} + 3x^2 + 2x^{-3} + 0
\]
\[
= 20x^{4 + \sqrt{2}} + 3\sqrt{2}x^2 + \frac{2}{x^3}
\]

b. \( h(x) = \frac{4}{5\sqrt{x}} - 11\sqrt{x^3} \)
\[
= \frac{4}{5} x^{-\frac{1}{2}} - 11 x^{\frac{3}{4}}
\]
\[
h''(x) = \frac{4}{5} \cdot \frac{1}{2} x^{-\frac{1}{2}} - 11 \cdot \frac{3}{4} x^{\frac{1}{4}}
\]
\[
= -\frac{4}{15} \frac{1}{\sqrt{x^4}} - \frac{33}{4} \frac{1}{\sqrt{x}}
\]

4. Suppose that for a certain function \( p(x) \), the line tangent to the graph of \( p \) at \( x=3 \) has equation \( y - 5 = 4(x - 3) \). What is the equation of the line tangent to the graph of \( 7p(x) \) at \( x=3 \)?

The y-coords and the slopes along \( 7p(x) \) are found by multiplying the y-coords and the slopes of \( p(x) \) by 7.
Since the point (3,5) is on the graph of \( p(x) \), (3,35) is on the graph of \( 7p(x) \).
Since the slope at (3,5) on \( p(x) \) is 4, the slope at (3,35) is 4 \cdot 7 = 28.
Finally, the equation we seek is \( y - 35 = 28(x - 3) \).

5. What is the equation of the line tangent to the graph of \( k(x) = 3x^4 - 2x^3 + x + 10 \) at the point \( (2, k(2)) \)?

The point on the graph is \( (2, k(2)) = (2, 44) \).
To find the slope we need \( k'(2) \). Now, \( k'(x) = 12x^3 - 6x^2 + 1 \)
so \( k'(2) = 12 \cdot 8 - 6 \cdot 4 + 1 \)
\[
= 73
\]
Finally, the equation of the line is \( y - 44 = 73(x - 2) \).
6A. What is our limit definition of the derivative of \( f(x) \) at \( x = a \)?

\[
\lim_{h \to 0} \frac{f(a + h) - f(a)}{h}, \text{ provided this limit exists.}
\]

6B. The graph of \( f(x) = \ln(x) \) is shown here. As yet we do not have a "rule" for finding the derivative of log functions, so we have to make a table as \( h \to 0 \) of a certain expression. Make that table to discover what \( f'(2) \) is, using \( h = 0.1, 0.01, 0.0001 \). Show all your answers to seven places after the decimal point, and make the correct observation about what \( f'(2) \) is.

<table>
<thead>
<tr>
<th>( h )</th>
<th>( \frac{f(2+h) - f(2)}{h} ) (i.e., ( \frac{\ln(2+h) - \ln(2)}{h} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.4879016</td>
</tr>
<tr>
<td>0.01</td>
<td>0.4987542</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.4999875</td>
</tr>
</tbody>
</table>

It appears that as \( h \to 0 \), \( \frac{f(2+h) - f(2)}{h} \) approaches \( \frac{1}{2} \); we conclude \( f'(2) = \frac{1}{2} \).

6C. Now, what is the equation of the dotted line in the figure?

The dotted line is tangent at \( (2, f(2)) = (2, \ln 2) \).
We see in 6B that the slope of that line is \( \frac{1}{2} \). Thus our equation is

\[
y - \ln 2 = \frac{1}{2} (x - 2) \quad \text{(or \( y - 0.6931472... = \frac{1}{2} (x - 2) \))}
\]