Math 106 Sections C and D
Quiz 1 (30 points)

Name: ________________________________

Show all your work to receive full credit for a problem.

There are four questions. Questions are printed on the back of the page too.

1. (5 points) Evaluate the following integral without using the calculator. If you use the table of integrals, mention the formula number and the values of the constants you use to find the answer.

\[ \int t \ln(t^2) \, dt \]
2. (8 points) Evaluate the following integral exactly (without using the table of integrals or the calculator):

\[ \int_{1}^{2} 5x^2 e^{x^3} \, dx \]
3. (9 points) The graph of $f'(x)$ is shown below.

(a) Rank the following estimates of $\int_1^3 f(x) \, dx$ in increasing order: $L_{20}$, $R_{20}$, $M_{20}$, $T_{20}$. Explain briefly in words or with a sketch why your order is correct.

(b) Find the least value of $n$ for which the error bound theorem guarantees that $R_n$ approximates $\int_1^3 f(x) \, dx$ within $\pm 0.01$. 
4. (8 points) Consider the IVP \( y' = x - y \), \( y(2) = 5 \).

   (a) Use Euler’s method with two steps to estimate \( y(3) \).

   (b) Show that the exact solution of the IVP is \( y = 4e^{2-x} + x - 1 \).