Math 205A Quiz 03 page 1 January 25, 2008 NAME suggested solutions

1. Let $A = \begin{bmatrix} 7 & 12 & 2 & -17 \\ -5 & -9 & -1 & 13 \\ 2 & 1 & 3 & 0 \\ 3 & 2 & 4 & -1 \end{bmatrix}$, let $\mathbf{c} = \begin{bmatrix} 37 \\ -26 \\ 13 \\ 19 \end{bmatrix}$ and let $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$ be the corresponding columns of A.

1A: Write all the solutions of $A\mathbf{x} = \mathbf{c}$ in the form $\mathbf{p} + \mathbf{v}_h$ where $\mathbf{p}$ is a particular solution of $A\mathbf{x} = \mathbf{c}$ and $\mathbf{v}_h$ represents all solutions of the corresponding homogeneous equation. Write down any RREF matrix you used to help solve this problem.

By calculator, the RREF of $[A | \vec{c}]$ is $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 7 \\ 0 & 1 & -1 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ showing the solutions of $A\vec{x} = \vec{c}$ to be

$$\begin{cases} x_1 = 7 - 2x_3 - x_4 \\ x_2 = -1 + x_3 + 2x_4 \\ x_3 = \quad x_3 \text{ (free)} \\ x_4 = \quad x_4 \text{ (free)} \end{cases} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

1B: Find *another* particular solution and explain how you got it.

any choices of x_3 & x_4 above give (all) soln's of $A\vec{x} = \vec{c}$, so for example, take $x_3 = 1$ & $x_4 = 1$ to get $\vec{x} = \begin{bmatrix} 7 \\ -1 \\ 0 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 1 \\ 1 \end{bmatrix}$ ∴ this is another particular soln. There are infinitely more.

1C: Let $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$ be any arbitrary vector in $\mathbf{R}^4$. What conditions (if any) must b_1, b_2, b_3 and b_4 satisfy in order for $\mathbf{b}$ to be in the span of $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$? Use the method discussed in class, and again, write down any RREF matrix you used to help solve this problem.

We're asking, when does $x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 + x_4\vec{v}_4 = \vec{b}$ have a soln. We discussed how the underlying system of eqn's can be represented via a "super-augmented" matrix

$$\begin{array}{cccc|cccc} (x_1 & x_2 & x_3 & x_4 & b_1 & b_2 & b_3 & b_4) \end{array}$$
$$\left[\begin{array}{cccc|cccc} 7 & 12 & 2 & -17 & 1 & 0 & 0 & 0 \\ -5 & -9 & -1 & 13 & 0 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 & 0 & 0 & 1 & 0 \\ 3 & 2 & 4 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$ which by calculator has RREF $$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 1 & 0 & 0 & 2 & -1 \\ 0 & 1 & -1 & -2 & 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 22 & -17 \\ 0 & 0 & 0 & 0 & 0 & 1 & -17 & 13 \end{array}\right]$$

which tells us that as the matrix A is put into RREF, the column $\begin{bmatrix} b_1 \\ \vdots \\ b_4 \end{bmatrix}$ changes to $\begin{bmatrix} 2b_3 - b_4 \\ -3b_3 + 2b_4 \\ b_1 + 22b_3 - 17b_4 \\ b_2 - 17b_3 + 13b_4 \end{bmatrix}$ and thus the underlying system is CONSISTENT $\Leftrightarrow \begin{cases} 0 = b_1 + 22b_3 - 17b_4 \\ 0 = b_2 - 17b_3 + 13b_4 \end{cases}$

1D: Verify that the entries of $\mathbf{c}$ do indeed satisfy these conditions; show your work.

need $\begin{cases} 0 \stackrel{?}{=} 37 + 22 \cdot 13 - 17 \cdot 19 = 37 + 286 - 323 = 0 ✓ \\ 0 \stackrel{?}{=} -26 - 17 \cdot 13 + 13 \cdot 19 = -26 - 221 + 247 = 0 ✓ \end{cases}$

1E: Does $\mathbf{R}^4 = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$? *Explain* your answer.

<u>NO</u>, since if $\vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$ does not satisfy both these conditions, it will not be a <u>Linear Combination</u> of $\vec{v}_1, \vec{v}_2, \vec{v}_3$ & $\vec{v}_4$; ie, $\vec{b}$ will not be in span$\{\vec{v}_1, \ldots, \vec{v}_4\}$