

1. Let  $A = \begin{bmatrix} 7 & 12 & 2 & -17 \\ -5 & -9 & -1 & 13 \\ 2 & 1 & 3 & 0 \\ 3 & 2 & 4 & -1 \end{bmatrix}$ , let  $\mathbf{c} = \begin{bmatrix} 37 \\ -26 \\ 13 \\ 19 \end{bmatrix}$  and let  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ , and  $\mathbf{v}_4$  be the corresponding columns of  $A$ .

1A: Write all the solutions of  $A\mathbf{x} = \mathbf{c}$  in the form  $\mathbf{p} + \mathbf{v}_h$  where  $\mathbf{p}$  is a particular solution of  $A\mathbf{x} = \mathbf{c}$  and  $\mathbf{v}_h$  represents all solutions of the corresponding homogeneous equation. Write down any RREF matrix you used to help solve this problem.

1B: Find *another* particular solution and explain how you got it.

1C: Let  $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$  be any arbitrary vector in  $\mathbf{R}^4$ . What conditions (if any) must  $b_1, b_2, b_3$  and  $b_4$  satisfy in order for  $\mathbf{b}$  to be in the span of  $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ , and  $\mathbf{v}_4$ ? Use the method discussed in class, and again, write down any RREF matrix you used to help solve this problem.

1D: Verify that the entries of  $\mathbf{c}$  do indeed satisfy these conditions; show your work.

1E: Does  $\mathbf{R}^4 = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ ? *Explain* your answer.