Math 205B&C Quiz 01 page 1 01/01/09 Name *Suggested Soln.* Section: 8am 1:10 pm

1. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ -7 \\ 5 \end{bmatrix}$. Also, let $\mathbf{b} = \begin{bmatrix} -5 \\ 11 \\ -5 \end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix} 6 \\ -10 \\ 4 \end{bmatrix}$.

1A. In terms of the definition, explain what it means to ask, "is b a *linear combination* of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$?" Your answer should start with, "are there scalars $\alpha_1 \ldots$".

"Are there scalars α_1, α_2, and α_3 for which $\alpha_1\vec{v}_1 + \alpha_2\vec{v}_2 + \alpha_3\vec{v}_3 = \vec{b}$?"

1B. Now, determine if b actually *is* a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$. Explain how you decide.

We consider the corresponding augmented matrix $\left[\begin{array}{ccc|c} 1 & -1 & 2 & -5 \\ -3 & 1 & -7 & 11 \\ 2 & 0 & 5 & -5 \end{array}\right]$ which has RREF $\left[\begin{array}{ccc|c} 1 & 0 & 2.5 & 0 \\ 0 & 1 & 0.5 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

the system corresponding to this matrix is inconsistent, as $0\alpha_1 + 0\alpha_2 + 0\alpha_3 = 1$ can't be solved. Therefore $\vec{b}$ is <u>Not</u> a L.C of $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_3$.

1C. To determine if $\mathbf{c}$ is in the span of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is equivalent to solving what system of linear equations?

$$\begin{cases} x_1 - x_2 + 2x_3 = 6 \\ -3x_1 + x_2 - 7x_3 = -10 \\ 2x_1 + 5x_3 = 4 \end{cases}$$

1D. Find all solutions to the system produced in part (C).

the corresponding augmented matrix is $\left[\begin{array}{ccc|c} 1 & -1 & 2 & 6 \\ -3 & 1 & -7 & -10 \\ 2 & 0 & 5 & 4 \end{array}\right]$ which has RREF $\left[\begin{array}{ccc|c} 1 & 0 & 2.5 & 2 \\ 0 & 1 & 0.5 & -4 \\ 0 & 0 & 0 & 0 \end{array}\right]$; the soln's of the underlying systems are $x_1 = 2 - 2.5x_3$, $x_2 = -4 - 0.5x_3$ where x_3 is free.

1E. What specific solution do you get if you set any and all free variables in part (D) to the number 2? Verify that your solution works by evaluating the corresponding linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$

With $x_3 = 2$ we obtain $\begin{cases} x_1 = 2 - 2.5 \cdot 2 = 2 - 5 = -3 \\ x_2 = -4 - 0.5 \cdot 2 = -4 - 1 = -5 \end{cases}$

So the specific soln is $(-3, -5, 2)$. Check: Does $-3\begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} - 5\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 2 \\ -7 \\ 5 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 6 \\ -10 \\ 4 \end{bmatrix}$

Yes: $\begin{bmatrix} -3 + 5 + 4 = 6 \checkmark \\ 9 - 5 - 14 = 9 - 19 = -10 \checkmark \\ -6 + 10 = 4 \checkmark \end{bmatrix}$