NAME:

Show ALL your work CAREFULLY.

(a) Use long division and the method of partial fractions to evaluate

$$\int \frac{2x^3 + 6x^2 + 4x + 1}{x^2 + 3x + 2} \, dx.$$

Since the numerator is a polynomial of degree higher than that of the denominator, we first use long division and obtain

$$\frac{2x^3 + 6x^2 + 4x + 1}{x^2 + 3x + 2} = 2x + \frac{1}{x^2 + 3x + 2}.$$ 

Note that $x^2 + 3x + 2 = (x + 1)(x + 2)$ so by using the method of partial fractions we obtain

$$\frac{1}{x^2 + 3x + 2} = \frac{1}{x + 1} - \frac{1}{x + 2}.$$ 

Now,

$$\int \frac{2x^3 + 6x^2 + 4x + 1}{x^2 + 3x + 2} \, dx = \int 2x \, dx + \int \frac{1}{x + 1} \, dx - \int \frac{1}{x + 2} \, dx = x^2 + \ln|x + 1| - \ln|x + 2| + C.$$ 

(b) Consider the following given data of a function $f(x)$ on the interval $[0, 4]$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>1</td>
<td>5</td>
<td>17</td>
</tr>
</tbody>
</table>

Find TRAP(2) AND MID(1). Here the notation $(n)$ indicates that the interval $[0, 4]$ is to be divided into $n$ equal subintervals.

First, we find LEFT(2) and RIGHT(2).

LEFT(2) = $f(0) \cdot \Delta x + f(2) \cdot \Delta x = 1 \cdot 2 + 5 \cdot 2 = 12$.

RIGHT(2) = $f(2) \cdot \Delta x + f(4) \cdot \Delta x = 5 \cdot 2 + 17 \cdot 2 = 44$. It follows that

TRAP(2) = $\frac{12 + 44}{2} = 28$. Finally, MID(1) = $f(2) \cdot \Delta x = 5 \cdot 4 = 20$.

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