Math 205A Quiz 02 page 1 January 18, 2008 NAME Suggested SOL'N's

1. Consider the system of equations
$$\begin{aligned} x_1 + 2x_2 + 4x_3 &= 21 \\ 4x_1 + 7x_2 + 13x_3 &= 72 \\ 3x_1 + 4x_2 + 6x_3 &= 39. \end{aligned}$$

1A. Use your calculator to find the RREF of the augmented matrix corresponding to this system and write the answer here.

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & -3 \\ 0 & 1 & 3 & 12 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

1B. What does the answer to (1A) tell us the solution to the system is? (Answer using the standard notation developed in class)

$$\begin{cases} x_1 = -3 + 2x_3 \\ x_2 = 12 - 3x_3 \\ x_3 = \text{free} \end{cases}$$ (ie. x_3 can be chosen arbitrarily)

1C. Suppose the "4" in the *third* equation in the above system is replaced by a "5". What is the RREF of the augmented matrix corresponding to this *new* system?

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

1D. What is the solution of the *new* system described in part (1C)? Verify that it works!

$$\begin{cases} x_1 = 5 \\ x_2 = 0 \\ x_3 = 4 \end{cases}$$

verification:
Does $5 + 2\cdot 0 + 4\cdot 4 = 21$? $5 + 16 \stackrel{?}{=} 21$ ✓ yes
Does $4\cdot 5 + 7\cdot 0 + 13\cdot 4 = 72$? $20 + 52 \stackrel{?}{=} 72$ ✓ yes
Does $3\cdot 5 + 5\cdot 0 + 6\cdot 4 = 39$? $15 + 24 \stackrel{?}{=} 39$ ✓ yes!

note the "5"!

2. Let $\mathbf{v}_1 = \begin{bmatrix}1\\4\\3\end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix}2\\7\\4\end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix}4\\13\\6\end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix}21\\72\\39\end{bmatrix}$.

2A. If possible, express the vector $\mathbf{b}$ as a linear combinations of the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ in *two different ways*, or explain why this cannot be done.

The question asks us to find two different soln's to $x_1\begin{bmatrix}1\\4\\3\end{bmatrix} + x_2\begin{bmatrix}2\\7\\4\end{bmatrix} + x_3\begin{bmatrix}4\\13\\6\end{bmatrix} = \begin{bmatrix}21\\72\\39\end{bmatrix}$ if possible. We solved the corresponding system in (1A) above.

Letting $x_3 = 0$ yields $-3\vec{v}_1 + 12\vec{v}_2 + 0\vec{v}_3 = \vec{b}$; letting $x_3 = 1$ gives $-1\vec{v}_1 + 9\vec{v}_2 + 1\vec{v}_3 = \vec{b}$, and of course there are infinitely many different ways to express $\vec{b}$ as a L.C. of $\vec{v}_1, \vec{v}_2$ & $\vec{v}_3$.

2B. Is $\mathbf{c} = \begin{bmatrix}22\\73\\40\end{bmatrix}$ in the span of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$? Fully explain your answer. Write down any matrices which support your conclusion.

The vector $\vec{c}$ is in Span$\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ $\Leftrightarrow$ $x_1\vec{v}_1 + x_2\vec{v}_2 + x_3\vec{v}_3 = \vec{c}$ has a soln. However, the augmented matrix corresponding to this equation has RREF $\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$, showing the system corresponding to this vector equation is inconsistent, that is, it has no soln. So $\vec{c}$ is not in the span of $\vec{v}_1, \vec{v}_2, \vec{v}_3$.