NAME:

Show ALL your work CAREFULLY.

(a) Recall that the error committed by using the Right Hand Sum $R_n$ is less than or equal to $\frac{K_1(b-a)^2}{2n}$ where $|f'(x)| \leq K_1$ for some constant $K_1$. Use this result to give an upper bound for $\left| \int_1^e (\ln x \cos(x^2)) \, dx - R_{10} \right|$. Here, $f(x) = (\ln x) \cos(x^2)$.

Note that $f'(x) = \frac{1}{x} \cos(x^2) + (\ln x)(-\sin(x^2))(2x)$. Over the interval $[1, e]$, $|f'(x)| \leq |\frac{1}{x} \cos(x^2)| + |(\ln x)(-\sin(x^2))(2x)|$

$\leq 1 \cdot 1 + 1 \cdot 1 \cdot 2e = 1 + 2e.$

Take $K_1 = 1 + 2e$. Then, the error is bounded by $\frac{(1+2e)(e-1)^2}{20}$.

Remark: To get a better (smaller) $K_1$, one finds the maximum value for $|f'(x)|$ over the interval $[1, e]$.

(b) Consider the initial value problem

$$\frac{dy}{dx} = -xy$$

with $y(0) = 2$.

Estimate the value $y(1)$ (when $x = 1$) of the solution using Euler’s method with two steps with initial point $(0, 2)$. DO THIS BY HAND and show all your steps.

Using two steps, the interval $[0, 1]$ is divided into two each of which is of length 0.5. Thus, $\Delta x = 0.5$. The slope is given by $-xy$. At $x = 0$, $y_0 = 2$.

At $x = 0.5$,

$$y_{0.5} = y_0 + (-0)(2) \cdot \Delta x = 2.$$

Then, at $x = 1$,

$$y_1 = y_{0.5} + (-0.5)(2) \cdot \Delta x = 2 - 0.5 = 1.5.$$

Therefore,

$$y(1) \approx 1.5.$$

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