

1. By hand, find a matrix in RREF which is row equivalent to the following matrix. Show and label all your steps as we've done in class, for example, if you add 4 copies of row 5 to row 8, write " $r_8 \leftarrow r_8 + 4r_5$ "; if you swap rows 4 and 5, write "swap r_4 and r_5 ", etc. (you don't need the quotes) Use steps that make the work easy.

$$\begin{bmatrix} 40 & 1 & 2 & 2000 & 6042 \\ 0 & 0 & 0 & 10 & 30 \\ 1 & 0 & 0 & 50 & 151 \end{bmatrix} \begin{array}{l} r_2 \leftarrow r_2/10 \\ \text{swap } r_1 \& r_3 \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 50 & 151 \\ 0 & 0 & 0 & 1 & 3 \\ 40 & 1 & 2 & 2000 & 6042 \end{bmatrix} \begin{array}{l} r_3 \leftarrow r_3 - 40r_1 \\ \text{swap } r_2 \& r_3 \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 50 & 151 \\ 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\begin{array}{l} \text{swap } r_2 \& r_3 \\ \text{swap } r_2 \& r_3 \end{array} \begin{bmatrix} 1 & 0 & 0 & 50 & 151 \\ 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix} \begin{array}{l} r_1 \leftarrow r_1 - 50r_3 \\ \text{swap } r_2 \& r_3 \end{array} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

RREF

NOTES (1) the order of steps in producing the RREF is NOT unique. BUT the final RREF itself IS.

(2) Do NOT assume this is an augmented matrix; it is INCORRECT to go on to

2. Suppose an augmented matrix is in RREF and one of its rows is $0\ 0\ 0\ 0\ 1$.

2A: Can there be any rows below this one in that augmented matrix? Explain.

yes, it's possible, but any such rows would have to be all zeros, i.e. $0\ 0\ 0\ 0\ 0$

say $\begin{cases} x_1 = 1 \\ x_2 = (\text{etc}) \\ x_3 = \dots \\ \text{etc} \end{cases}$ (It might just be a matrix of coefficients for example)

2B: Does the underlying system of equations have no solutions, one, or infinitely many? Fully explain your answer.

No solns, because $0\ 0\ 0\ 0\ 1$ represents the equation

$0x_1 + 0x_2 + 0x_3 + 0x_4 = 1$; this cannot be satisfied by any values of x_1, \dots, x_4 since $0 \neq 1$.

3. Suppose the augmented matrix corresponding to some system of equations is row equivalent to

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 3 \\ 0 & 1 & 5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

3A: What are the solutions? (Express them in terms of any free variables)

$$\begin{cases} x_1 = 3 + 2x_3 \\ x_2 = 0 - 5x_3 \\ x_3 \text{ is free} \\ x_4 = 1 \end{cases} \text{ (so there are infinitely many solns as } x_3 \text{ can be chosen arbitrarily)}$$

3B: What are the pivot columns of this matrix? Use notation like c_1, c_2, c_3 etc.

c_1, c_2 and c_4 (columns with leading 1's in the RREF)