

NAME:

SECTION: (circle one) 11:00-11:55 12:05-1:00

Math 105 - Exam 1 - October 7, 2005

Instructions: Show all of your work and circle your final answers. Calculators are allowed, but notes and books are not.

1. (24 pts. - 8 pts. each)

(a) Consider the differential equation $\frac{dy}{dt} = \frac{y-2}{t+1}$. Is $y = 2t + 4$ a solution? Justify your answer.

(b) Give all possible solutions of F to the differential equation $F' = 4x^2 + 3e^2 + 2^x$.

(c) If $f(x) = (\sin x)\sqrt{x} + \frac{x}{\ln x}$, find $f'(x)$.

2. (10 pts.) State the formal limit definition of $f'(a)$.

3. (24 pts. - 8 pts. each) Consider the function $f(x) = x^2 - 4x$.

(a) Calculate the average rate of change of $f(x)$ on the interval $[3, 4]$.

(b) Using your definition in Problem 2, calculate $f'(3)$.

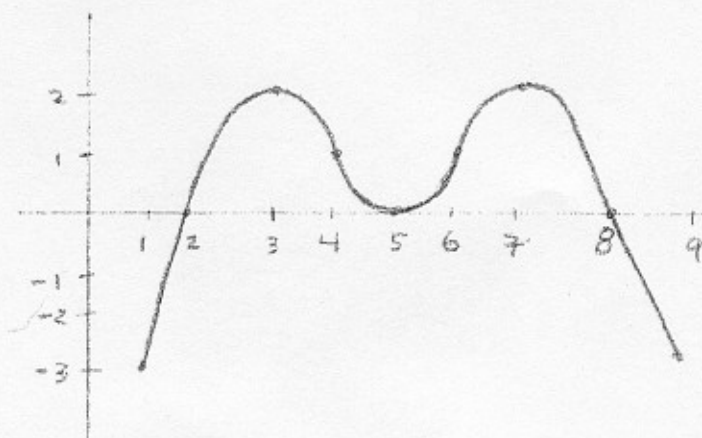
(c) Find the equation of the tangent line on the graph of $y = f(x)$ at the point where $x = 3$.

4. (10 pts. - 5 pts. each)

(a) Find the domain of the function $g(x) = \sqrt{4 - x^2}$.

(b) Suppose the range of the function $f(x)$ is $[-1, 1)$. Calculate the range of the function $f(x+3) + 4$.

5. (32 pts. - 4 pts. each) Let $F(x)$ be an antiderivative of $f(x)$ (in other words, $F'(x) = f(x)$). The graph of $y = f(x)$ is given below. Find the following:



- (a) Where is $F(x)$ increasing?
- (b) Where is $f(x)$ increasing?
- (c) Where is $f'(x)$ increasing?
- (d) Where does F have (a) stationary point(s)?
- (e) Where is F concave up?
- (f) Where does F have (a) local maximum point(s)?
- (g) Where does F have (an) inflection point(s)?
- (h) Which is largest: $F(3)$, $F(6)$, or $F(8)$?

6. BONUS (5 pts): Calculate $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x}$ (and justify your answer).

Grading : do not write in this area

1	2	3	4	5	B	TOTAL