A Sample Beamer Presentation

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January 6, 2012
1. What Can Happen at a Critical Point?

2. What Does $g'(c) > 0$ Mean?

3. Further Work
The Usual Suspects

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If that's what you think, then you are ... (notice that we’re giving you time to reconsider!) ... wrong.
A Counterexample

Consider the function

\[ f(x) = \begin{cases} 
  x^2 \sin(1/x), & \text{if } x \neq 0 \\
  0, & \text{if } x = 0 
\end{cases} \]

Let’s see what \( f'(0) \) is.
Finding $f'(0)$

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Since \(-h \leq h \sin(1/h) \leq h\) and \(\lim_{h \to 0} (-h) = \lim_{h \to 0} (h) = 0\), the Squeeze Theorem says \(f'(0) = 0\).
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What Really Happens at \( x = 0 \)?

But \( f(x) \) oscillates wildly as \( x \to 0 \), so even though \( f'(0) = 0 \), \( f \) has neither max, min, nor inflection point at \( x = 0 \).
What Really Happens at $x = 0$?

But $f(x)$ oscillates wildly as $x \to 0$, so even though $f'(0) = 0$, $f$ has neither max, min, nor inflection point at $x = 0$.

$y = f(x), y = x^2, y = -x^2$
1 What Can Happen at a Critical Point?

2 What Does $g'(c) > 0$ Mean?

3 Further Work
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But what does “increasing at $x = c$” really mean?
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A Reasonable Definition

A function $g$ is *increasing at* $x = c$ if there is an open interval $I = (c - \delta, c + \delta)$ such that
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A function $g$ is increasing at $x = c$ if there is an open interval $I = (c - \delta, c + \delta)$ such that if $x_1, x_2 \in I$, then $x_1 < x_2 \Rightarrow g(x_1) < g(x_2)$. 
Our Function with a Slight Twist

Let’s modify our function to

\[ g(x) = \begin{cases} 
0.5x + x^2 \sin(1/x), & \text{if } x \neq 0 \\
0, & \text{if } x = 0 
\end{cases} \]

Using the definition of derivative as before, we will find that \( g'(0) = 0.5 \).
However, $g(x)$ still oscillates enough as $x \to 0$ that there is no open interval containing $x = 0$ that satisfies our definition of $g$ increasing at $x = 0$ even though $g'(0) > 0$. 
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However, $g(x)$ still oscillates enough as $x \to 0$ that there is no open interval containing $x = 0$ that satisfies our definition of $g$ increasing at $x = 0$ even though $g'(0) > 0$.

\[ y = g(x), \quad y = x^2 + 0.5x, \quad y = x^2 - 0.5x \]
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3. Further Work
The function $f(x)$ introduced earlier has other interesting properties, one of which is the fact that while $f'(0)$ exists, $f'(x)$ is discontinuous at $x = 0$.

We leave it to you to work this out for yourself and to explore this interesting function further.

Thank you for your attention today.