Lesson Eight: Over/Under and Boxes

Overline and Underline
In some cases, we like to underline important notions, such as cardinality. A common notation for the cardinality of the set \( A \) is \( \overline{A} \).

Overbrace and Underbrace
Can’t remember those pesky exponent rules? Happens to all of us. \(^1\) Suppose \( m, n \in \mathbb{N} \). Then we have the following

\[
a^m a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ times}} \underbrace{a \cdot a \cdot \ldots \cdot a}_{n \text{ times}} = (a \cdot a \cdot \ldots \cdot a) \underbrace{a \cdot a \cdot \ldots \cdot a}_{m+n \text{ times}} = a^{m+n}
\]

We all know that \( e^w = 1 + w + \frac{w^2}{2!} + \frac{w^3}{3!} + \ldots \) for all \( w \in \mathbb{R} \). But what happens if we’re so reckless as to let \( w = ix \), where \( i = \sqrt{-1} \in \mathbb{C} \)? Let’s see.

\[
e^{ix} = 1 + ix + \frac{(ix)^2}{2!} + \frac{(ix)^3}{3!} + \frac{(ix)^4}{4!} + \frac{(ix)^5}{5!} + \ldots
\]

\[
= 1 + ix - \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \ldots
\]

\[
= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + i \left( \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots \right)
\]

\[
= \cos x + i \sin x
\]

Boxes
A consequence of the above formula is the following, which relates five of the most important numbers in mathematics and thus deserves its own box.

\[
e^{i\pi} + 1 = 0
\]

Exercise Eight: Over/Under and Boxes
Some colleges to which one might apply are the following.

Yale, Colby, Bowdoin, Wisconsin, Harvard, Bates

safety schools top choices

More on those easily forgotten exponent rules. As before, we let \( m, n \in \mathbb{N} \).

\[
a^m = \underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ times}} = a^{m-n}
\]

\[
(a^m)^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ times}} \ldots \underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ times}} \underbrace{a \cdot a \cdot \ldots \cdot a}_{m \text{ times}} = a^{mn}
\]

If \( A \) and \( B \) are finite sets, then \( A \cap B = \emptyset \iff \overline{A} + \overline{B} = \overline{A \cup B} \)

\(^1\)Well, not to me, of course.