Lesson Three: Proclamations and Proofs

**Definition.** An integer $m$ is **even** if $m = 2j$ where $j$ is an integer.

**Theorem 1.** The sum of any two even integers is an even integer.

**Proof.** Suppose $m$ and $n$ are even integers.
   By the definition of even, $m = 2j$ and $n = 2k$ where $j$ and $k$ are integers.
   Therefore, $m + n = 2j + 2k = 2(j + k)$.
   Since the integers are closed under addition, $j + k$ is also an integer.
   So, $m + n$ is twice another integer $(j + k)$, meaning $m + n$ is even, as desired. □

**Theorem 2.** The equation $x^n + y^n = z^n$ has no non-zero integer solutions for $n > 2$.

**Proof.** I have a marvellous proof of this, but the page is too small to contain it. □

**Theorem 3.** The number 8675309 is prime.

**Proof.** Just ask Jenny. □

**Definition.** A mathematician is a device for turning coffee into theorems. [attributed to Paul Erdős]

**Note.** Begin laughing now.

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Exercise Three: Proclamations and Proofs

**Theorem 1.** The product of any two even integers is an even integer.

**Proof.** [Try this on your own before looking at the solutions.] □

**Definition.** An integer $k$ is **odd** if $k = 2j + 1$ where $j$ is an integer.

**Theorem 2.** The sum of any two odd integers is an even integer.

**Proof.** [Try this on your own before looking at the solutions.] □

**Theorem 3.** Every even integer greater than two is the sum of two primes.

**Proof.** [Let us know right away if you find a proof to this one!] □