Name: ________________________________

Show all work, clearly and legibly, to receive full credit. Correct spelling, organization of your solution, and proper use of mathematical notation all count. You may use a calculator, but no notes, books, or other resources. Good luck!

1.) (10 pts.)

a.) (5 pts.) What are the hypotheses of the MVT? Which hypotheses are satisfied for the function \( f(x) = |x| \) on the interval \([-1, 2]\)? Based on your answer so far: should we expect \( f(x) \) to satisfy the conclusion of the MVT?

b.) (5 pts.) For this part, you do not need to use the function \( f(x) \) from part (a). Here: state the conclusion of the MVT, and illustrate the conclusion using a graph. Your graph should include a function that satisfies the hypotheses of the MVT. Label your graph to show what each part of the equality in the MVT’s conclusion refers to.
2.) (15 pts.) The graph below is $g'$. NOTE: this is NOT a graph of $g$.

\[ \text{Graph of } g' \]

a.) (5 pts.) Suppose that $g(10) = 7$. Find an equation of the line tangent to $g$ when $x = 10$.

b.) (5 pts.) Is $g(0) < g(3.5)$? Justify your answer.

c.) (5 pts.) Sketch a graph of $g''$ on the axes below.

\[ \text{Graph axes with } g'' \]
3.) (15 pts.) The graph below is $g'$. Use this graph to answer the questions below. NOTE: this is NOT a graph of $g$.

![Graph of $g'$](image)

a.) (3 pts.) At which $x$-value(s), or on what interval(s), does $g$ have a local maximum?

b.) (3 pts.) At which $x$-value(s), or on what interval(s), is $g$ increasing?

c.) (3 pts.) At which $x$-value(s), or on what interval(s), is $g''$ least?

d.) (3 pts.) At which $x$-value(s), or on what interval(s), is $g''$ negative?

e.) (3 pts.) Sketch a graph of $g$ on the axes below, setting $g(0) = -3$. 

![Axes for sketch of $g$](image)
4.) (15 pts.) Compute the following limits. Show all necessary steps.

a.) (5 pts.) \( \lim_{x \to \infty} \frac{\sin x}{x} \)

b.) (5 pts.) \( \lim_{x \to 0} \frac{7x - \sin x}{2x} \)

c.) (5 pts.) \( \lim_{x \to 2} \frac{f(x)}{g(x)} \), where \( f(x) \) and \( g(x) \) are given in the graphs below.

![Graph of f(x)](image)

![Graph of g(x)](image)
5.) (15 pts.) Compute the following derivatives. Show enough steps to make clear what you are doing.

a.) (5 pts.) \( v'(t) \), if \( v(t) = \log_3 \left( 4 \tan \left( e^{5t^2} \right) \right) \)

b.) (5 pts.) \( m'(x) \), if \( m(x) = 2010^x + 2010^{-x} + x^{2010} + \left[ \sin(2010x) \right]\left[ \ln(2010x) \right] \)

c.) (5 pts.) \( h'(-1) \), if \( h(x) = \frac{f(x)}{g(x)} \), \( f(-1) = 2 \), \( g(-1) = 3 \), \( f'(-1) = 4 \), and \( g'(-1) = 5 \)
6. (15 pts.) Consider the graph of $f(x)$ below, and define $g(x) = \int_0^x f(t) \, dt$.

a.) (5 pts.) Compute $T_6$ to approximate the area under the curve of $f(t)$ for $0 \leq t \leq 6$. Do this both by shading in the graph accordingly, and by computing the areas of the trapezoids and adding them as signed area.

b.) (5 pts.) Compute $g(6)$ exactly using geometry. The curves are both semicircles.

c.) (5 pts.) Compute $g'(x)$. (This should be a formula, not a number.)
7.) (15 pts.) Suppose a forest fire spreads in a circle with radius changing at a rate of 5 feet per minute. When the radius reaches 200 feet, at what rate is the area of the burning region increasing?

In your response, include a labeled picture and appropriate Calculus techniques.