Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).

- Circle or otherwise indicate your final answers.

- Please keep your written answers brief; be clear and to the point. I will take points off for rambling and for incorrect or irrelevant statements.

- This test has 10 problems and is worth 100 points. It is your responsibility to make sure that you have all of the pages!

- Good luck!
1. (15 points) The following is a graph of \( f(x) \) on the interval \([-4,5]\).

a. (2 pts) Where is \( f'(x) \) positive?

b. (2 pts) Where is \( f'(x) \) decreasing?

c. (2 pts) Where is \( f''(x) \) negative?

d. (2 pts) Where is \( f'(x) \) negative?

e. (2 pts) Does the hypothesis of the Mean Value Theorem hold on the interval \([-4,2]\)? Explain.

f. (2 pts) Does the conclusion of the Mean Value Theorem hold on the interval \([-4,2]\)? Explain.

g. (3 pts) Does the conclusion of the Intermediate Value Theorem hold on the interval \([-4,3]\)? Explain.
2. (10 points)

\[
\begin{array}{c|cccccc}
  x & f(x) & g(x) & j(x) & f'(x) & g'(x) & j'(x) \\
  \hline
  -2 & -1 & 1 & -1 & 3 & 2 & 1 \\
  -1 & 1 & 3 & 2 & -1 & 3 & -2 \\
  0 & 2 & 1 & 1 & -2 & -2 & 2 \\
  1 & 3 & 1 & -1 & -1 & 3 & 1 \\
  2 & -2 & 2 & 1 & 3 & 2 & 3 \\
  3 & -1 & 1 & -1 & 1 & -2 & 2 \\
\end{array}
\]

a. (5 pts) \( H(x) = \sqrt{f(x^2)} + \ln(j(x)) \). Find \( H'(1) \).

b. (5 pts) \( F(x) = \frac{e^xg(x)}{f(x)^3} \). Find \( F'(2) \).

3. (8 points) Find the derivative of the following functions.

a. (4 pts) \( g(s) = \sqrt[3]{s^3} + \frac{5}{s} + 2\cos(s) + \arctan(e^{3s} + 5\pi) + \ln(3) \)

b. (4 pts) \( y = \frac{(2x + 4)^3(x^2 - 2)^3e^{-3x}}{(x - 4)^3(5x^3 - 1)^2} \) (Use logarithmic differentiation)
4. (7 points) For the equation $x^3 + y^3 = \ln(xy) - 1$ use implicit differentiation to find $\frac{dy}{dx}$.

5. (8 points) Solve the following. Only use L'Hôpital's rule when appropriate. Show your work!!
   a. (4 pts) $\lim_{x \to 0} \frac{\sqrt[3]{4-x^2} - 2}{x}$
   b. (4 pts) $\lim_{x \to 0} x^x$

6. (7 points) Find differentiable functions $f(x)$ and $g(x)$ such that $\lim_{x \to 5} f(x) = 0$ and $\lim_{x \to 5} g(x) = 0$ and
   a. (3 pts) $\lim_{x \to 5} \frac{f(x)}{g(x)} = 10$
   b. (4 pts) $\lim_{x \to 5} \frac{f(x)}{g(x)} = \infty$
7. (8 points) All edges of a cube are expanding at the same rate. The surface area is changing at a rate of 12 cm\(^2\)/second when each edge measures 3cm. Determine how fast the volume of the cube is changes when the edges are 3cm. (This is a two step problem)

8. (10 points)
   a. (4 pts) Determine the antiderivative of \(2t + \frac{2t}{1 + 4t^4}\). Show your check.

   b. (2 pts) Find the derivative of \(\ln(32 - 5x^2)\).

   c. (4 pts) Use the FTC to determine \(\int_0^2 \frac{x}{32 - 5x^2} dx\).
9. (10 points) A right triangle is formed in the first quadrant by the \( x- \) and \( y- \)axes and a line through the point (2,3). Find the vertices of the triangle so that its area is minimum. (Hint: The area of the triangle should be a function of the slope of the line. You don’t know the slope.)
10. \((17\ \text{points})\) Let \(f(x) = 3x^2 + 1\).

a. \((4\ \text{pts})\) Estimate the area \(\int_1^4 f(x)dx\) using 3 rectangles and left-hand sums.

\[\int_1^4 f(x)dx\]

b. \((3\ \text{pts})\) Determine \(\int_1^4 f(x)dx\) using the FTC.

c. \((10\ \text{pts})\) Use infinite Riemann sums \(\left(\int_a^b f(x)dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x\right)\) to find \(\int_1^4 f(x)dx\).

\[
\sum_{i=1}^{n} 1 = n, \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2
\]