1. Find a parameterization $f$ and corresponding region $R$ for the helicoid shown. Note the helicoid wraps around the $z$-axis three times, and the points on the outer helix boundary are 2 units from the $z$-axis; finally, the bottom is 1 unit below the $x$-axis and the top is 2 units above it.

$$f(s, t) = (t \cos s, t \sin s, \frac{s}{2\pi})$$

where $s \in [-2\pi, 4\pi]$ and $t \in [0, 2]$ 

(i.e. $R = [-2\pi, 4\pi] \times [0, 2]$)

2. Recall one way to parameterize the unit sphere is

$$f(s, t) = (\cos(s) \cos(t), \cos(s) \sin(t), \sin(s))$$

for $(s, t)$ in the rectangle $R = [-\pi/2, \pi/2] \times [0, 2\pi]$.

Find a subset $A$ of $R$ such that the parameterization $f$ restricted to $A$ yields the part of the sphere shaded in below. The lines connecting the poles are called meridians, or lines of longitude. They are 9 degrees apart on this figure. The bold meridian passes through $(1, 0, 0)$. Also shown is the “equator”, which is the unit circle in the $xy$ plane. The lines running parallel to it are called “parallels”, or lines of latitude. On this figure they are 6 degrees apart. Give your answer in terms of radians.

The shaded region covers from $5 \times 6^\circ = 30^\circ = \pi/6$ north latitude to $7.5 \times 6^\circ = 45^\circ = \pi/4$.

It also covers from $-5 \times 9^\circ = 45^\circ + 45^\circ$ from “west to east”. However, we need correspondingly values in $0$ to $2\pi$, thus

$-45^\circ$ to $45^\circ$, or $-\pi/4$ to $\pi/4$ becomes $-\pi$ to $\pi$.

Together with $0$ to $\pi/4$; thus $s \in [-\pi/4, \pi/4]$

and $t \in \left[\frac{\pi}{6}, \frac{\pi}{4}\right]$ 

and the required subset is $\left[\frac{\pi}{6}, \frac{\pi}{4}\right] \times \left[\frac{\pi}{6}, \frac{\pi}{4}\right]$.

3. Consider the surface $M$ having the parameterization given by $f(s, t) = (s^2, t^2, s + t)$ over the region in the $st$ plane bounded by $s = 1$, $s = 2$ and the curves $t = s^2$ and $t = 6 - s$. Set up the double integral (with appropriate limits on the integrals) which represents the surface area of $M$. Simplify the integrand as much as possible.

4. Let $M$ be as in (3). Let $g(x, y, z) = y + xz$. Set up the integral $\int_M g \, dS$; simplify the integrand as much as possible.

This adds a factor of $g(f(s, t))$ to the integral in 3, that is:

$$\int_{s=1}^{s=2} \int_{t=5}^{t=5} \left( t^3 + s^2(s+t) \right) \sqrt{9t^4 + 4s^2 + 36s^2 t^4} \, dt \, ds.$$