Read directions carefully and show all your work. Partial credit will be assigned based upon the correctness, completeness, and clarity of your answers.

1. (5 pts) At one moment, one bicyclist is 4 miles east of an intersection traveling west toward the intersection at the rate of 9 miles/hour. At the same time a second bicyclist is 3 miles south of the intersection traveling south away from the intersection at a rate of 10 miles/hour. Is the distance between the cyclists increasing or decreasing at that moment? At what rate?

\[ \frac{dx}{dt} = -9 \text{ mph, } \quad \frac{dy}{dt} = 10 \text{ mph} \]

Find \( \frac{dz}{dt} \)

\[
x^2 + y^2 = z^2 \quad \left\rightarrow \text{ implicitly differentiate with respect to } t \right\]

\[
2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}
\]

\[
\frac{dz}{dt} = \frac{x \frac{dx}{dt} + y \frac{dy}{dt}}{z}
\]

\[
= \frac{(4 \text{ mi})(-9 \text{ mi/hr}) + (3 \text{ mi})(10 \text{ mi/hr})}{5 \text{ mi}}
\]

\[
= -1.2 \text{ mph}
\]

The distance is decreasing at 1.2 mph.

2. (5 pts) Consider \( f(x) = \sqrt{x} \). You are not allowed to use a calculator for this problem.

(a) What does the Extreme Value Theorem say about \( f \) on the interval \([0, 1]\)?

\( f(x) = \sqrt{x} \) is continuous on \([0, 1]\) since \( f \) is defined for all values of \( x \) in \([0, 1]\).

EVT says \( f \) achieves both a global max and global min on \([0, 1]\).

(b) Does the Intermediate Value Theorem hold for \( f \) on \([0, 4]\) and \( y = 3 \)? Explain your answer.

\( f \) is continuous on \([0, 1]\)

\( f(0) = 0 \) and \( f(4) = 2 \)

But \( y = 3 \) is not between \( f(0) \) and \( f(4) \), so the conditions of the IVT are not satisfied. Therefore, the IVT does not apply in this case.