NAME:

Show ALL your work CAREFULLY.

(a) Use the Ratio Test to determine, if possible, whether the following infinite series converges or diverges.

\[ \sum_{n=1}^{\infty} \frac{n^2}{2^n} \]

Since \( a_n = \frac{n^2}{2^n} \), we have

\[
\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = \lim_{n \to \infty} \frac{(n+1)^2/2^{n+1}}{n^2/2^n} = \lim_{n \to \infty} \left( \frac{n+1}{n} \right)^2 \cdot \frac{2^n}{2^{n+1}} = \frac{1}{2} \lim_{n \to \infty} \left( \frac{n+1}{n} \right)^2 = \frac{1}{2}.
\]

By the Ratio Test, the limit \( \frac{1}{2} < 1 \) so the series \( \sum_{n=1}^{\infty} \frac{n^2}{2^n} \) converges.

(b) Determine whether the following alternating series converges conditionally, converges absolutely, or diverges. If it converges, find lower and upper bounds for the limit. Justify your answer.

\[ \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{\sqrt{k}} \]

The corresponding series of absolute values is \( \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}} \) which diverges by the \( p \)-test (with \( p = \frac{1}{2} \)). Since the sequence \( \{ \frac{1}{\sqrt{k}} \} \) is decreasing to 0, the Alternating Series Test asserts that the series \( \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{\sqrt{k}} \) converges. Hence, we conclude that \( \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{\sqrt{k}} \) converges conditionally.

The first and second partial sums are \( S_1 = \frac{1}{\sqrt{1}} = 1, S_2 = 1 - \frac{1}{\sqrt{2}} \) so we have

\[ 1 - \frac{1}{\sqrt{2}} < \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{\sqrt{k}} < 1. \]

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