Math 105 - Quiz 8 - November 27, 2006

Instructions: Show all of your work and circle your final answers. Calculators are allowed, but notes and books are not.

1. (12 pts.) A cubical block of ice is melting in such a way that it always keeps the shape of a cube. Suppose the volume of the ice is decreasing at a rate of 3 cm³ per minute. At what rate is the length of an edge changing when each edge is 10 cm in length?

\[ V = x^3 \]

Given: \( \frac{dV}{dt} = -3 \text{ cm}/\text{min} \).

Find \( \frac{dx}{dt} \) when \( x = 10 \text{ cm} \).

\[ V = x^3 \]

So \( \frac{dV}{dt} = \frac{d}{dt}(x^3) \),

So \( \frac{dV}{dt} = 3x^2 \cdot \frac{dx}{dt} \),

So \( -3 = 3(10)^2 \cdot \frac{dx}{dt} \),

So \( \frac{-1}{100} = \frac{dx}{dt} \) Hence, side length decreases at a rate of \( \frac{1}{100} \text{ cm}/\text{min} \).

2. (8 pts.) Let \( f(x) = x^2 \). What does the mean value theorem say about \( f(x) \) on the interval \([1, 2]\)? Find all suitable values of \( c \).

Since \( f \) is continuous on \([1, 2]\) and differentiable on \((1, 2)\),

there is an \( x \)-value \( c \) in \((1, 2)\) where \( f'(c) = \frac{f(2) - f(1)}{2 - 1} = \frac{4 - 1}{1} = 3 \).

\( f'(x) = 2x \), so \( f'(c) = 2c \), so \( 2c = 3 \rightarrow c = \frac{3}{2} \).

At \( c = \frac{3}{2} \), slope = 3 (= average slope on \([1, 3]\) .)