1. Suppose that a square of side length \( s \) and area \( A \) is changing size with \( \frac{dA}{dt} = 60 \text{ cm}^2 \) per second at all times \( t \).

1A. At what rate is the length of the side changing at the moment when the area is 25 cm\(^2\)? Include the correct units in your answer.

\[ \text{so: } \frac{dA}{dt} = 2s \frac{ds}{dt} \]

\[ \Rightarrow 60 \text{ cm}^2/\text{sec} = 2.5 \text{ cm} \cdot \frac{ds}{dt}; \quad \frac{dA}{dt} = ds/dt \]

\[ \text{when } A = 25 \text{ cm} \Rightarrow s = 5 \text{ cm} \]

\[ \Rightarrow \frac{dA}{dt} = 6 \text{ cm/sec} \]

1B. When the side reaches 10 cm, what will the rate of change in its length be at that instant?

\[ \text{again, } \frac{dA}{dt} = 2s \frac{ds}{dt} \] but now \( s = 10 \text{ cm} \) instead of 5;

\[ \text{get: } 60 = 2 \cdot 10 \cdot \frac{ds}{dt} \Rightarrow \frac{ds}{dt} = \frac{60}{20} = 3 \text{ (cm/sec)} \]

2. Suppose \( f \) is a polynomial. Then \( f \) is continuous on \([a, b]\) and differentiable on \((a, b)\) for any endpoints \( a \) and \( b \). Given \( a \) and \( b \) then, the Mean Value Theorem says there must be a \( c \) in \((a, b)\) for which \( f'(c) = \frac{f(b) - f(a)}{b - a} \) WHAT EXPRESSION? 

\[ f'(c) = \frac{f(b) - f(a)}{b - a} \]

2A. Suppose in fact that \( f(x) = x^3 \) and \([a, b] = [1, 3]\). Find the \( c \) that’s guaranteed to exist by the MVT. Show all your work and write \( c \) to at least five places after the decimal point.

\[ \text{so we need } f(c) = f(3) - f(1) \]

\[ \Rightarrow 3c^2 = \frac{27 - 1}{3 - 1} = \frac{26}{2} = 13 \]

\[ 3c^2 = 13 \]

\[ c^2 = \frac{13}{3} \]

\[ c = \pm \sqrt{\frac{13}{3}} = \pm 2.081665... \]

... we need \( c \in (1, 3) \) : \[ c = 2.081665... \]

3. The graph of a function \( f(x) \) is made of straight lines and semicircles is shown at the bottom of the page. Find each of the following integrals.

\[ \int_2^4 f(x) \, dx \]

\[ \int_6^8 f(x) \, dx \]

\[ \int_4^2 f(x) \, dx \]

\[ \int_4^2 f(x) \, dx \]

\[ \int_4^5 10f(x) \, dx = 10 \int_4^5 f(x) \, dx = 10 \cdot \frac{3}{2} = 15 \]

This little \( \frac{1}{4} \pi \)

\[ \text{two \Delta's are involved here and they contribute } \frac{3}{2} - \frac{1}{2} = 1 \]

The area of \( f(x) \) from 2 to 4.

\[ \int_{-2}^2 f(x) \, dx \]

\[ = \int_{-2}^2 (6 + \pi) \, dx \]

\[ = \left[ (6 + \pi) \right]_{-2}^2 \]

\[ = 2(6 + \pi) \]

Since there are 2 pieces, with the area from 8 to 10 and they are below the axis, we get

\[ = -2(1 - \frac{\pi}{4}) \]

\[ = -2 + \frac{\pi}{2} \]