1. (6) Determine if the series is convergent or divergent. \[ \sum_{n=1}^{\infty} \frac{\ln(n)}{n^4} \]

Since \( \ln(n) \leq n^d \) (eventually) for \( d > 0 \), we have \( a_n = \frac{\ln(n)}{n^4} \leq \frac{n^d}{n^d} = \frac{1}{n^{4-d}} \).

The way we have the inequality, we need to choose a \( d \) value that gives a \( b_n \) that produces a convergent series. Any value with \( 0 < d < 3 \) will work. Taking \( d = 1 \) gives \( a_n = \frac{\ln(n)}{n^4} \leq \frac{1}{n^3} \).

By the PST, \( \sum_{n=1}^{\infty} \frac{1}{n^3} \) is convergent. So by DCT, \( \sum_{n=1}^{\infty} \frac{\ln(n)}{n^2} \) is convergent.

2. (4) Determine if the series is Convergent or Divergent. \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{3^n n!} \]

Try the Ratio Test

\[
\lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \to \infty} \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n} = \lim_{n \to \infty} \frac{3}{n+1} = 0 < 1
\]

So by the Ratio Test, the series \( \sum_{n=1}^{\infty} \frac{(-1)^n}{3^n n!} \) is Absolutely Convergent (A.C.). This means that the series is Convergent.