1. (6 points) Compute \( \int_R (2x^2 + 3y) \, dA \) for the region \( R \) in the first quadrant bounded by \( x = 0, y = 4, \) and \( y = x^2 \). \( R \) is also shown below.

\[
\int_0^2 \int_{x^2}^4 (2x^2 + 3y) \, dy \, dx = \int_0^2 \left[ 2x^2y + \frac{3}{2}y^2 \right]_{y=x^2}^4 \, dx
\]

\[
= \int_0^2 \left( 8x^2 + 24 \right) - \left( 2x^4 + \frac{3}{2}x^4 \right) \, dx
\]

\[
= \frac{8}{3}x^3 + 24x - \frac{2}{5}x^5 - \frac{3}{10}x^5 \bigg|_0^2
\]

\[
= \frac{64}{3} + 48 - \frac{64}{5} - \frac{48}{5}
\]

\[
= \frac{64}{3} + \frac{144}{3} - \frac{112}{15} = \frac{208}{3} - \frac{112}{5}
\]

\[
= \frac{1040}{15} - \frac{336}{15} = \frac{704}{15}
\]

This integral can also be computed if the order of integration reversed. See the bottom of the next page.

2. (6 points) Reverse the order of integration for the following integral. (Do NOT evaluate the integral.)

\[
\int_0^2 \int_{\sqrt{xy}}^{x^2} f(x, y) \, dy \, dx
\]

When \( y = x \), \( x = \sqrt{xy} \) \( x = \sqrt{y} \) \( x^2 = 8y \) \( \frac{x^2}{8} = y \)

\[
\int_0^4 \int_0^{\sqrt{8x}} f(x, y) \, dy \, dx
\]

when \( y = x^2 \)

\[
= \frac{704}{15}
\]

over
3. (4 points) Let $R$ be the region shown. Let the values of the single variable integrals be as given.

\[
\begin{align*}
\int_0^1 g(x,1) \, dx &= 1 \\
\int_0^{0.75} g(x,0.75) \, dx &= 2 \\
\int_0^{0.25} g(x,0.75) \, dx &= 3 \\
\int_0^{0.5} g(x,0.5) \, dx &= 4 \\
\int_0^{0.75} g(x,0.25) \, dx &= 5 \\
\int_0^{0.25} g(x,0.25) \, dx &= 6 \\
\int_0^1 g(x,0) \, dx &= 7
\end{align*}
\]

Each of these represents the area of a cross section.

Use some of the values of the integrals above to approximate

\[
\iint_R g(x,y) \, dA = \int_0^1 \left[ \int_0^{1-y} g(x,y) \, dx \right] dy.
\]

Some of the integrals in the list give us this boxed value at certain $y$-values.

\[
\begin{align*}
\int_0^1 \int_0^{1-y} g(x,y) \, dx \, dy &= \int_0^{1-0} g(x,0) \, dx \cdot (.25) + \int_0^{1-.25} g(x, .25) \, dx \cdot (.25) \\
&+ \int_0^{1-.25} g(x, .5) \, dx \cdot (.25) + \int_0^{1-.75} g(x, .75) \, dx \cdot (.25) \\
&= \frac{1}{4} \left( 7 + 5 + 4 + 3 \right) = \frac{19}{4}
\end{align*}
\]

Problem (done using the other order of integration)

\[
\begin{align*}
\int_0^1 \int_0^{y^2} (2x^2 + 3y) \, dx \, dy &= \int_0^1 \left[ \frac{2}{3} x^3 + 3y x \right]_{x=0}^{x=y^2} \, dy \\
&= \int_0^1 \left( \frac{2}{3} y^3 + 3y^3 \right) \, dy \\
&= \frac{4}{15} y^{5/2} + \frac{6}{8} y^{5/2} \bigg|_0^1 = \frac{4}{15} (2^5) + \frac{18}{15} (2^5) = \frac{22}{15} \cdot 32 = \frac{704}{15}
\end{align*}
\]