1. Consider the path parameterized by \( f(t) = (t^3/3 - t, \sin t) \) for \( t \in [-\pi, \pi] \).

1A) Make a nice sketch of this path in the window \([-8, 8] \times [-1, 1]\). Use \( \text{Tstep}=0.1 \) Make an excellent facsimile of your calculator's drawing on the axes provided. The \( \text{Xsc1} \) and \( \text{Ysc1} \) values are 1 and 0.5 respectively.

![Path Sketch]

1B) Set up (but don’t try to evaluate) the integral which represents the arc length of this path.

\[
\int_{-\pi}^{\pi} \| \dot{f}(t) \| \, dt = \int_{-\pi}^{\pi} \| (t^2-1, \cos t) \| \, dt
\]

\[
= \int_{-\pi}^{\pi} \sqrt{(t^2-1)^2 + (\cos t)^2} \, dt
\]

1C) Use your calculator’s numeric integration program to find to two places after the decimal point, the length of this path. (NB: you may need to switch your calculator from Param back to Func after drawing the path for part (1A) before you can use the built-in \( \int f(x) \, dx \) feature).

\[ \approx 18.18 \]

2) A “fence” is built over the path parameterized by \( f(t) = (t, t^2 - 9) \) for \( t \in [0, 3] \). The height of the fence over any point \((x, y)\) on the path is \( 2x + 3y + 27 \). Set up (but don’t evaluate) the integral that represents the area of one side of the fence.

\[
\int_{0}^{3} u(f(t)) \| \dot{f}(t) \| \, dt \quad \text{where} \quad \dot{f}(t) = (1, 2t)
\]

\[
= \int_{0}^{3} (2t + 3(t^2 - 9) + 27) \sqrt{1^2 + (2t)^2} \, dt
\]

\[
= \int_{0}^{3} (2t + 3t^2) \sqrt{1 + 4t^2} \, dt
\]

3) The path parameterized by \( f(t) = (t, t^2 - 9) \) for \( t \in [0, 3] \) runs through the vector field \( \mathbf{F}(x, y) = (y, x) \). If this field represents a force, find the work done by the force on an object moving on that path with the given parameterization. Show all your work, from the integral you need to set up, to its evaluation.

\[
= \int_{0}^{3} \mathbf{F}(f(t)) \cdot f'(t) \, dt
\]

\[
= \int_{0}^{3} (t^2 - 9, t) \cdot (1, 2t) \, dt
\]

\[
= \int_{0}^{3} t^2 - 9 + 2t^2 \, dt
\]

\[
= \int_{0}^{3} 3t^2 - 9 \, dt
\]

\[
= t^3 - 9t \bigg|_{0}^{3} = 27 - 27 = 0
\]