Correct answers accompanied by incorrect or incomplete work will not receive full credit. Good Luck!

1. Compute \[ \sum_{i=4}^{\infty} \frac{3}{ii^4} = \frac{3}{ii^4} + \frac{3}{ii^5} + \frac{3}{ii^6} + \ldots \]

   this is a geometric series w/ ratio \( \frac{1}{i^4} \) and \( \left| \frac{1}{i^4} \right| < 1 \)

   So \[ \text{sum} = \frac{15.4 \text{ term}}{1 - \frac{1}{i^4}} = \frac{3/i^4}{i^4 - 1/i^4} = \frac{3}{i^4(i^4 - 1)} \]

2. Does the following series converge or diverge? Justify your answer (be sure to identify which convergence test you use).

   \[ 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \ldots \]

   diverges b/c it is the harmonic series

   \[ \sum_{k=1}^{\infty} \frac{1}{k} \text{ diverges by the p-test w/ } p = 1 \]

3. Does the following series converge or diverge? Justify your answer (be sure to identify which convergence test you use).

   \[ \sum_{j=1}^{\infty} \frac{1}{\sqrt{j} + e^j} \]

   \( \mathcal{O} \) \[ \frac{1}{\sqrt{j} + e^j} \leq \frac{1}{e^j} \] \( \mathcal{O} \) \[ \sum_{j=1}^{\infty} \frac{1}{e^j} \text{ is a geometric} \]

   series w/ \( \left| r \right| = \left| \frac{1}{e^j} \right| < 1 \), so it converges.

   Thus \( \mathcal{O} \) and \( \mathcal{O} \) imply \[ \sum_{j=1}^{\infty} \frac{1}{\sqrt{j} + e^j} \text{ converges by the comparison test.} \]
4. The following are graphs of \( a(x) = \frac{1}{x^2 + 1} \), \( b(x) = \frac{1}{(x - 2)^2 + 1} \), and \( c(x) = \frac{e^{\sin x}}{x^2} \).

The three integrals \( \int_1^{\infty} a(x) \, dx \), \( \int_1^{\infty} b(x) \, dx \), and \( \int_1^{\infty} c(x) \, dx \) all converge.

(a) Can you use the integral test to determine whether or not \( \sum_{k=1}^{\infty} \frac{1}{k^2 + 1} \) converges? Why or why not? Yes, \( \frac{1}{x^2 + 1} \) is decreasing, positive, continuous, and integrable on \([1, \infty)\).

(b) Can you use the integral test to determine whether or not \( \sum_{k=1}^{\infty} \frac{1}{(k-2)^2 + 1} \) converges? Why or why not? Yes, \( \frac{1}{(k-2)^2 + 1} \) is decreasing, positive, continuous, and integrable on \([2, \infty)\).

So \( \sum_{k=2}^{\infty} \frac{1}{(k-2)^2 + 1} \) converges, thus \( \sum_{k=1}^{\infty} \frac{1}{(k-2)^2 + 1} \) must converge.

(c) Can you use the integral test to determine whether or not \( \sum_{k=1}^{\infty} \frac{e^{\sin k}}{k^2} \) converges? Why or why not? No, there is no \( x \)-value after which \( \frac{e^{\sin x}}{x^2} \) is decreasing.

Since \( \frac{e^{\sin x}}{x^2} \) is not decreasing, we can't use the integral test.