1. Is \( \vec{x} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} \) an eigenvector of \( A = \begin{bmatrix} -3 & 1 \\ -3 & 5 \end{bmatrix} \)? Explain.

\[ \begin{bmatrix} -3 & 1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 17 \end{bmatrix} \]

\( \begin{bmatrix} 1 \\ 17 \end{bmatrix} \) is not a multiple of \( \begin{bmatrix} 1 \\ 4 \end{bmatrix} \), \( \begin{bmatrix} 1 \\ 4 \end{bmatrix} \) is not an eigenvector.

2. Find the characteristic equation for \( A \) and determine the eigenvalues of \( A \).

\[ \begin{vmatrix} -3 - \lambda & 1 \\ -3 & 5 - \lambda \end{vmatrix} = (-3 - \lambda)(5 - \lambda) + 3 = \lambda^2 - 2\lambda - 12 = 0 \]

\( \lambda = \frac{2 \pm \sqrt{4 + 48}}{2} = 1 \pm \sqrt{13} \)

3. Is \( \lambda = 1 \) an eigenvalue of \( B = \begin{bmatrix} 3 & 2 \\ 3 & 8 \end{bmatrix} \)? Explain.

\( B - I = \begin{bmatrix} 2 & 2 \\ 3 & 7 \end{bmatrix} \sim \begin{bmatrix} 10 \\ 01 \end{bmatrix} \) which has a trivial nullspace. \( \lambda = 1 \) is not an eigenvalue of \( B \).

4. The eigenvalues of \( C \) are \( \lambda = 1, 5 \).

\( C = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix} \)

If possible, determine \( P \) and \( D \) such that \( C = PDP^{-1} \).

\( \lambda = 1 \Rightarrow \begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & -1 \\ -1 & -2 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{Basis} = \{ \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \} \)

\( \lambda = 5 \Rightarrow \begin{bmatrix} -3 & 2 & -1 \\ 1 & 2 & -1 \\ -1 & -2 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{Basis} = \{ \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \} \)

\( P = \begin{bmatrix} -2 & 1 & -1 \\ 0 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \)

\( D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix} \)